Series and Sequence

Arithmetic (common difference "d")	Geometric (Common Ratio "r")
$t_n = a + (n-1)d$	$t_n = ar^{n-1}$
$S_n = \frac{n}{2}(2a + (n-1)d)$	$S_n = \frac{a(r^n - 1)}{r - 1}$
$S_n = \frac{n}{2}(a + t_n)$	$\boxed{S_n = \frac{a}{1 - r}}$

1. Identify as Arithmetic or Geometric and then find the number of terms "n"

a) -5, -2, 1, 4, …103.

Arithmetic

$a = -5$

$d = t_2 - t_1$

$= -2 - (-5) = 3$

$t_n = 103$

$t_n = a + (n-1)d$

$103 = -5 + (n-1)3$

$103 = -5 + 3n - 3$

$103 = 3n - 8$

$111 = 3n$

$\boxed{37 = n}$

b) 2, 6, 18, …486.

Geometric

$a = 2$

$r = \frac{t_2}{t_1} = \frac{6}{2} = 3$

$t_n = 486$

$t_n = ar^{n-1}$

$486 = (2)(3)^{n-1}$

$243 = 3^{n-1}$

$3^5 = 3^{n-1}$

$5 = n - 1$

$\boxed{6 = n}$

c) $\frac{1}{4}$, $\frac{1}{2}$, 1, … 64.

Geometric

$a = 1/4$

$r = \frac{1/2}{1/4} = 2$

$t_n = 64$

$t_n = ar^{n-1}$

$64 = \left(\frac{1}{4}\right)(2)^{n-1}$

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$

$8 = n - 1$

$\boxed{9 = n}$

As it aged, a maple tree produced sap according to the pattern shown in the table below.

Year	2001	2002	2003	2004
Sap (Litres)	$t_1 = 60.000$	$t_2 = 57.000$	$t_3 = 54.150$	$t_4 = 51.4425$

a) Does the data follow an arithmetic or geometric pattern? $r = \frac{57}{60} = 0.95$

b) Write down a formula for t_n?

$t_n = ar^{n-1}$

$t_n = 60(0.95)^{n-1}$

c) Assuming the pattern continues, how long will it take for the sap production to be approximately 17.5L? (Solve for "n")

$17.5 = 60(0.95)^{n-1}$ ← t_n

$0.29\overline{16} = (0.95)^{n-1}$

$(0.95)^{24} = (0.95)^{n-1}$

$24 = n-1$

$25 = n$

∴ It will take 25 yrs.

d) If the tree lives for a very long time approximately how much sap will it produce from 2001 on? (Infinite Geometric Series)

$S_n = \frac{a}{1-r} = \frac{60}{1-0.95} = \frac{60}{0.05} = \boxed{1200\text{L}}$

∴ It will produce 1200L of sap from 2001 on.

9. An emergency measures organization uses a fan-out system to alert staff. The executive officer who initiates the action makes five calls. Each of these people in turn makes five calls, and so on. How many people have been alerted after six calls if the executive officer is considered the first call?

$a = 1$
$r = 5$
$n = 6$
$S_n = ?$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1(5^6 - 1)}{5 - 1}$$

$$= \frac{15625 - 1}{4}$$

$$= \frac{15624}{4}$$

$$= 3906$$

∴ 3906 people have been alerted.