Limits Review

1. Evaluate the following limits if they exist.

(a) $\lim_{x\to -5} \dfrac{x^2-25}{x^2+5x}$

$\lim_{x\to -5} \dfrac{(x-5)(x+5)}{x(x+5)}$

$\lim_{x\to -5} \dfrac{(x-5)}{x} = \dfrac{-10}{-5} = \boxed{2}$

(b) $\lim_{x\to \infty} \dfrac{2x^2-x-6}{(3x^2-1)^2}$

$\lim_{x\to \infty} \dfrac{2x^2-x-6}{9x^4-6x^2+1}$

$= \boxed{0}$

(c) $\lim_{x\to 1} \dfrac{(x+3)^3-64}{x-1}$

$\lim_{x\to 1} \dfrac{((x+3)-4)((x+3)^2+4(x+3)+16)}{x-1}$

$= 16+16+16$

$= \boxed{48}$

(d) $\lim_{x\to 5} \dfrac{(\sqrt{x+4}-3)}{(x-5)}$ $\dfrac{(\sqrt{x+4}+3)}{(\sqrt{x+4}+3)}$

$\lim_{x\to 5} \dfrac{x+4-9}{(x-5)(\sqrt{x+4}+3)}$

$= \boxed{\dfrac{1}{6}}$

2. Given the function … $f(x)=\begin{cases}(x+3)^2 & \text{if } x<-2\\ -x-1 & \text{if } -2\le x<1\\ 1 & \text{if } x=1\\ (x-2)^2-3 & \text{if } x>1\end{cases}$

Using the three conditions for continuity examine *f(x)* for any points of discontinuity. Draw a sketch of *f(x)* and list any point(s) of discontinuity

$(x+3)^2$

x	y
-2	1
-3	0
-4	1
-5	4

$-x-1$

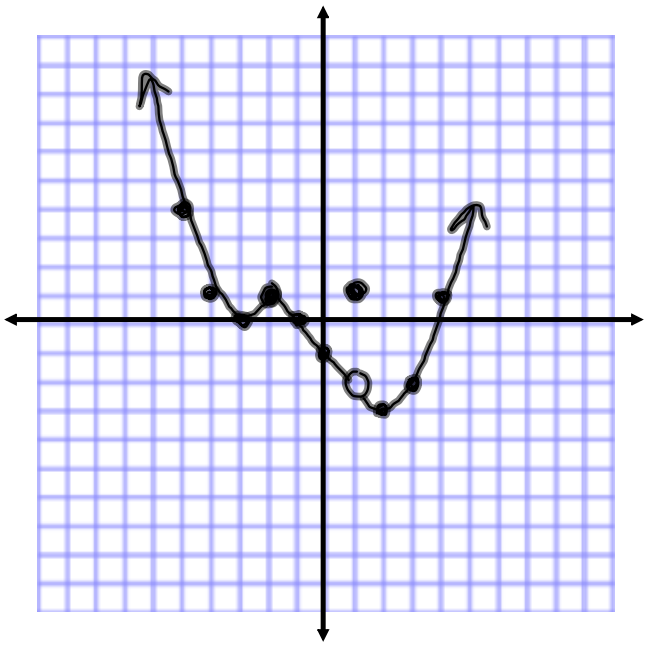
x	y
-2	1
-1	0
0	-1
1	-2

1

x	y
1	1

$(x-2)^2-3$

x	y
1	-2
2	-3
3	-2
4	1



Discontinuous at x=1

4. Differentiate the following functions using the *limit definition of the derivative:*

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

a) $f(x)=x^2+4x+2$

$$F(x+h)=(x+h)^2+4(x+h)+2$$

$$F(x+h)=x^2+2xh+h^2+4x+4h+2$$

$$f'(x)=\lim_{h\to 0}\frac{x^2+2xh+h^2+4x+4h+2-(x^2+4x+2)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{2xh+h^2+4h}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{h(2x+h+4)}{h}=\boxed{2x+4}$$

Slope of the tangent

5. Find the equation of the tangent line to the curve at the given point.

a) $y = (x^2 + 1)^2$ at $(-1, 4)$

$y = x^4 + 2x^2 + 1$

① Find derivative:

$y' = 4x^3 + 4x$

② Find Slope (Sub in "x")

$y' = 4(-1)^3 + 4(-1)$

$y' = -4 - 4$

$y' = \boxed{-8}$ → Slope "m"

③ Find the equation

$y - y_1 = m(x - x_1)$

$y - 4 = -8(x + 1)$

$y - 4 = -8x - 8$

$\boxed{8x + y + 4 = 0}$

7. Find the derivative: *Express answers with positive exponents!*

a) $f(x) = 3x^5 + \sqrt[3]{x}$

$f(x) = 3x^5 + x^{1/3}$

$f'(x) = 15x^4 + \frac{1}{3}x^{-2/3}$

$f'(x) = 15x^4 + \frac{1}{3x^{2/3}}$

b) $f(x) = \sqrt[5]{x^2}$

$f(x) = x^{2/5}$

$f'(x) = \frac{2}{5}x^{-3/5}$

$f'(x) = \frac{2}{5x^{3/5}}$