

Series and Sequence

Arithmetic
(common difference "d")

$$t_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric
(Common Ratio "r")

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{a}{1 - r}$$

$$-1 < r < 1$$

1. Identify as Arithmetic or Geometric and then find the number of terms "n"

a) -5, -2, 1, 4, ... 103.

b) 2, 6, 18, ... 486.

c) $\frac{1}{4}, \frac{1}{2}, 1, \dots 64.$

Arithmetic

$$\begin{aligned} a &= -5 & t_n &= a + (n-1)d \\ d &= 3 & 103 &= -5 + (n-1)(3) \\ t_n &= 103 & 108 &= 3n - 3 \\ & & 111 &= 3n \\ & & \boxed{37} &= n \end{aligned}$$

b) 2, 6, 18, ... 486.

Geometric

$$\begin{aligned} a &= 2 & t_n &= ar^{n-1} \\ r &= 3 & 486 &= \frac{2}{2} \cdot \frac{(\cancel{3})^{n-1}}{\cancel{3}} \\ t_n &= 486 & & \\ & & 243 &= 3^{n-1} \\ & & 3^5 &= 3^{n-1} \\ & & 5 &= n-1 \\ & & \boxed{6} &= n \end{aligned}$$

c) $\frac{1}{4}, \frac{1}{2}, 1, \dots 64.$

Geometric

$$\begin{aligned} a &= \frac{1}{4} & t_n &= ar^{n-1} \\ r &= 2 & 64 &= \frac{(\cancel{4})}{\frac{1}{4}} \cdot \frac{(\cancel{2})^{n-1}}{\cancel{4}} \\ t_n &= 64 & & \\ & & 256 &= 2^{n-1} \\ & & 2^8 &= 2^{n-1} \\ & & 8 &= n-1 \\ & & \boxed{9} &= n \end{aligned}$$

As it aged, a maple tree produced sap according to the pattern shown in the table below.

Year	2001	2002	2003	2004
Sap (Litres)	$t_1 = \underline{\underline{60.000}}$ $a = 60$	$t_2 = 57.000$	$t_3 = 54.150$	$t_4 = 51.4425$

a) Does the data follow an arithmetic or geometric pattern?

$$r = \frac{57}{60} = \frac{54.15}{57} = 0.95 \quad \text{geometric}$$

b) Write down a formula for t_n ?

$$t_n = ar^{n-1}$$

$$t_n = 60(0.95)^{n-1}$$

c) Assuming the pattern continues, how long will it take for the sap production to be approximately 17.5L?

$$t_n = 17.5L. \quad \frac{17.5}{60} = \frac{60(0.95)^{n-1}}{60}$$

$$n = ?$$

$$0.291\bar{6} = 0.95^{n-1}$$

$$0.95^{24} = 0.95^{n-1}$$

$$24 = n-1$$

$$\boxed{25 = n}$$

d) If the tree lives for a very long time approximately how much sap will it produce from 2001 on? (Infinite Geometric Series)

$$S_n = \frac{a}{1-r} \quad * \quad -1 < r < 1 \quad \leftarrow \begin{array}{l} r=0.95 \\ \text{satisfies} \end{array}$$

$$S_n = \frac{60}{1-0.95}$$

$$= \frac{60}{0.05}$$

$$\boxed{= 1200 L}$$

8. A computer software company formed a committee of 5 people to spread the word about a new feature. Each person on the committee emailed 3 individuals (cycle 1) who were each asked to email 3 more people (cycle 2). This pattern continued and new individuals were contacted at each cycle (i.e. nobody received two emails).

a) What is the value of t_1 ? $5 \times 3 = 15$ $a = 15$ [1]

b) Write down a formula for t_n . $t_n = ar^{n-1} = (15)(3)^{n-1}$ [1]
 $a = 15$ $r = 3$

c) On which cycle were 32 805 people contacted? [2]

d) Find the total number of people contacted in 4 cycles. [2]

$$c) \quad \frac{32805}{15} = \frac{(15)(3)^{n-1}}{15}$$

$$2187 = 3^{n-1}$$

$$3^7 = 3^{n-1}$$

$$7 = n - 1$$

$$\boxed{8 = n}$$

$$d) \quad S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_4 = \frac{15(3^4 - 1)}{3 - 1}$$

$$S_4 = \frac{1200}{2}$$

$$\boxed{S_4 = 600}$$