

Questions from Homework

$$\begin{aligned} \textcircled{2} \quad y &= 2x^3 - 5x^2 - 4x + 3 \\ &= 2(-1)^3 - 5(-1)^2 - 4(-1) + 3 \\ &= -2 - 5 + 4 + 3 \\ &= 0 \end{aligned}$$

if $x = -1$ makes $y = 0$ then $(x+1)$ is a factor.

$$\begin{array}{r} \text{Decomp.} \quad \begin{matrix} -6x - 1 = 6 \\ -b + c = -7 \end{matrix} \\ \downarrow \\ (x+1)(2x^2 - 7x + 3) \\ (x+1)(2x^2 - 6x - 1x + 3) \\ (x+1)[2x(x-3) - 1(x-3)] \\ \boxed{(x+1)(2x-1)(x-3)} \end{array}$$

$$\begin{array}{r} \text{X+1} \overline{) 2x^3 - 5x^2 - 4x + 3} \\ \underline{-(2x^3 + 2x^2)} \\ -7x^2 - 4x \\ \underline{-(-7x^2 - 7x)} \\ 3x + 3 \\ \underline{-(3x + 3)} \\ 0 \end{array}$$

$$\textcircled{3} \quad y = (x+3)^{\overset{a=1}{\downarrow}}(x-2)(x-3)$$

Roots: $x = -3, -3, 2, 3$

Quartic with a positive stretch factor Starts in Q2 and ends in Q1.

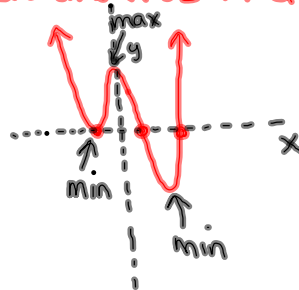
c) App. Local Max ($x = -0.5$)

$$y = (x+3)^2(x-2)(x-3)$$

$$y = (2.5)^2(-2.5)(-3.5)$$

$$y = 54.7$$

$$(0.5, 54.7)$$



d) App. Local Min ($x = 2.5$)

$$y = (x+3)^2(x-2)(x-3)$$

$$y = (5.5)^2(0.5)(-0.5)$$

$$y = -7.6$$

$$(2.5, -7.6)$$

Limits Review

1. Evaluate the following limits if they exist.

(a) $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x^2 + 5x}$

$$\lim_{x \rightarrow -5} \frac{(x-5)\cancel{(x+5)}}{x\cancel{(x+5)}}$$

$$\lim_{x \rightarrow -5} \frac{(x-5)}{x} = \frac{-10}{-5} = \boxed{2}$$

(b) $\lim_{x \rightarrow \infty} \frac{2x^2 - x - 6}{(3x^2 - 1)^2}$

$$\lim_{x \rightarrow \infty} \frac{2x^2 - x - 6}{9x^4 - 6x^2 + 1} = \boxed{0}$$

(c) $\lim_{x \rightarrow 1} \frac{(x+3)^3 - 64}{x-1}$

$$\lim_{x \rightarrow 1} \frac{(\cancel{x+3}-4)(\cancel{x+3}^2 + 4\cancel{x+3} + 16)}{\cancel{x-1}}$$

$$= 16 + 16 + 16 = \boxed{48}$$

(d) $\lim_{x \rightarrow 5} \frac{(\sqrt{x+4}-3)(\sqrt{x+4}+3)}{(x-5)(\sqrt{x+4}+3)}$

$$\lim_{x \rightarrow 5} \frac{\cancel{x+4}-9}{(\cancel{x-5})(\sqrt{x+4}+3)}$$

$$= \boxed{\frac{1}{6}}$$

2. Given the function ...

$$f(x) = \begin{cases} (x+3)^2 & \text{if } x < -2 \\ -x-1 & \text{if } -2 \leq x < 1 \\ 1 & \text{if } x = 1 \\ (x-2)^2 - 3 & \text{if } x > 1 \end{cases}$$

Using the three conditions for continuity examine $f(x)$ for any points of discontinuity. Draw a sketch of $f(x)$ and list any point(s) of discontinuity

$(x+3)^2$

x	y
-2	1
-3	0
-4	1
-5	4

$-x-1$

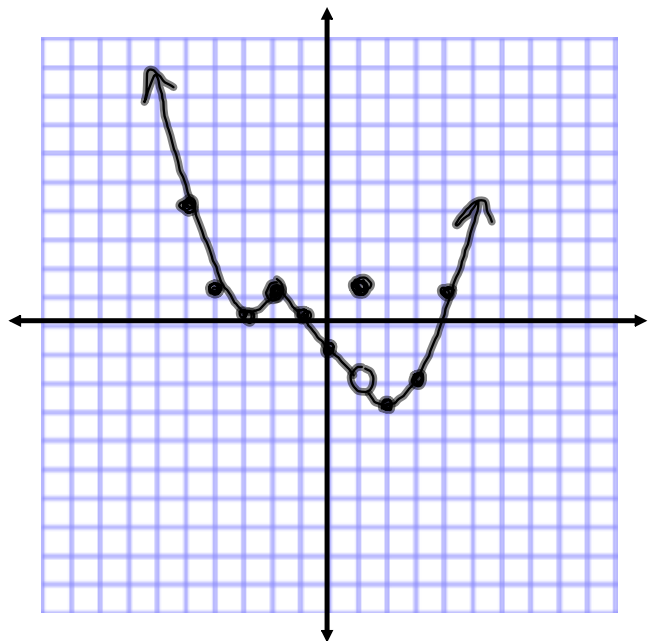
x	y
-2	1
-1	0
0	-1
1	-2

1

x	y
1	1

$(x-2)^2 - 3$

x	y
1	-2
2	-3
3	-2
4	1



Discontinuous at $x=1$

4. Differentiate the following functions using the *limit definition of the derivative*:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

a) $f(x) = x^2 + 4x + 2$

$$F(x+h) = (x+h)^2 + 4(x+h) + 2$$

$$F(x+h) = x^2 + 2xh + h^2 + 4x + 4h + 2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{4x} + 4h + \cancel{2} - (\cancel{x^2} + \cancel{4x} + \cancel{2})}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 4h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}(2x + \underline{h} + 4)}{\cancel{h}} = \boxed{2x + 4}$$

Slope of the tangent

5. Find the equation of the tangent line to the curve at the given point.

a) $y = (x^2 + 1)^2$ at $(-1, 4)$

$$y = x^4 + 2x^2 + 1$$

① Find derivative:

$$y' = 4x^3 + 4x$$

② Find Slope (Sub in "x")

$$y' = 4(-1)^3 + 4(-1)$$

$$y' = -4 - 4$$

$$y' = \boxed{-8} \rightarrow \text{Slope "m"}$$

③ Find the equation

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -8(x + 1)$$

$$y - 4 = -8x - 8$$

$$\boxed{8x + y + 4 = 0}$$

7. Find the derivative: *Express answers with positive exponents!*

a) $f(x) = 3x^5 + \sqrt[3]{x}$

$$f(x) = 3x^5 + x^{1/3}$$

$$f'(x) = 15x^4 + \frac{1}{3}x^{-2/3}$$

$$f'(x) = 15x^4 + \frac{1}{3x^{2/3}}$$

b) $f(x) = \sqrt[5]{x^2}$

$$f(x) = x^{2/5}$$

$$f'(x) = \frac{2}{5}x^{-3/5}$$

$$f'(x) = \frac{2}{5x^{3/5}}$$

Homework.

$$\textcircled{1} \text{ a) } \lim_{x \rightarrow 0} \frac{\overset{(x+a)}{\cancel{x+a}} \cdot \underset{\cancel{1}}{a} - \frac{1}{\cancel{1}} (x+a)}{x(x+a)} \quad \text{CD: } (x+a)$$

$$\lim_{x \rightarrow 0} \frac{a - x - a}{x(x+a)}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{-x}}{\cancel{x}(x+a)} = \boxed{\frac{-1}{a}}$$