

## **Questions from Homework**

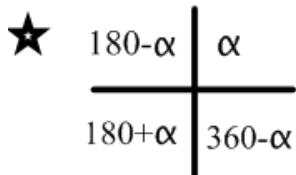
# Complex Numbers

## Convert to Polar coordinates

Find the radius  $r$ , using the Pythagorean relationship  $r = \sqrt{x^2 + y^2}$

Find the related angle,  $\alpha$ , using  $\alpha = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

Find the angle,  $\theta$ , by determining the quadrant in which the terminal arm is located and using the related angle.



Remember from last semester

The polar coordinates are  $(r, \theta)$

## Convert to Rectangular coordinates

$$x = r \cos \theta$$

$$y = r \sin \theta$$

# De Moivre's Theorem

$$(rcis\theta)^n = r^n cis n\theta$$

1. Simplify the following expressions. Express solution in the form  $a + bi$ .

$$(a) 3i^7 - i^{10} + (2i)^5$$

$$3i^7 - i^{10} + 32i^5$$

$$3(i^4)(i^3) - (i^8)(i^2) + 32(i^4)i$$

$$-3i + 1 + 32i$$

$$\boxed{1 + 29i}$$

$$(b) \frac{(1+i)(2+3i)}{-3+2i}$$

$$\frac{2 + 5i + 3i^2}{-3+2i}$$

$$\frac{(-1+5i)(-3-2i)}{(-3+2i)(-3-2i)}$$

$$\frac{3 - 13i - 10i^2}{9 - 4i^2}$$

$$\frac{13 - 13i}{13}$$

$$\boxed{1 - i}$$

3. Use polar coordinates and De Moivre's Theorem to evaluate the following expression:

(Express solution in the form  $a + bi$ )

$$\frac{(-\sqrt{3} + i)^4 (2 - 2i)^6}{(-1 - i\sqrt{3})^{10}}$$

$$\begin{aligned} a &= -\sqrt{3} \quad b = 1 \\ r &= \sqrt{(-\sqrt{3})^2 + 1^2} \quad \alpha = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) \quad \text{Quad } 3 \\ r &= \sqrt{3+1} \quad \theta = 180 - 30^\circ \\ r &= 2 \quad \alpha = 30^\circ \quad \theta = 150^\circ \\ & \boxed{2 \text{ cis } 150^\circ} \end{aligned}$$

$$\begin{aligned} a &= 2 \quad b = -2 \\ r &= \sqrt{(2)^2 + (-2)^2} \quad \alpha = \tan^{-1}\left(\frac{-2}{2}\right) \quad \text{Quad } 4 \\ r &= \sqrt{4+4} \quad \alpha = \tan^{-1}(1) \quad \theta = 360 - 45^\circ \\ r &= 2\sqrt{2} \quad \alpha = 45^\circ \quad \theta = 315^\circ \\ & \boxed{2\sqrt{2} \text{ cis } 315^\circ} \end{aligned}$$

$$\begin{aligned} a &= -1 \quad b = -\sqrt{3} \\ r &= \sqrt{1+3} \quad \alpha = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) \quad \text{Quad } 3 \\ r &= 2 \quad \alpha = \tan^{-1}(\sqrt{3}) \quad \theta = 180 + 60^\circ \\ & \alpha = 60^\circ \quad \theta = 240^\circ \\ & \boxed{2 \text{ cis } 240^\circ} \end{aligned}$$

$$\frac{(\text{2 cis } 150^\circ)^4 (2\sqrt{2} \text{ cis } 315^\circ)^6}{(2 \text{ cis } 240^\circ)^{10}}$$

$$\frac{(16 \text{ cis } 600^\circ)(512 \text{ cis } 1890^\circ)}{1024 \text{ cis } 2400^\circ}$$

$$\frac{8192 \text{ cis } 2490^\circ}{1024 \text{ cis } 2400^\circ}$$

$\boxed{8 \text{ cis } 90^\circ}$  as a polar number

$$\begin{aligned} a &= 8 \cos 90^\circ & b &= 8 \sin 90^\circ \\ &= 8(0) & &= 8(1) \\ &= 0 & &= 8 \end{aligned}$$

$$\boxed{0 + 8i}$$

as a  
rectangular  
number.