Functions Toolkit #2
Solutions

1 a) $\dfrac{4}{(x-6)(x+5)} - \dfrac{2}{(x+5)(x+3)}$

$\dfrac{4x+12-2x+12}{(x-6)(x+5)(x+3)}$

$\boxed{\dfrac{2x+24}{(x-6)(x+5)(x+3)},\ x \neq -5,-3,6}$

b) $\dfrac{2x}{3x+5} + \dfrac{x}{3x^2-6x+5x-10}$

$\dfrac{2x}{(3x+5)} + \dfrac{x}{(3x+5)(x-2)}$

$\dfrac{2x^2-4x+x}{(3x+5)(x-2)}$

$\boxed{\dfrac{2x^2-3x}{(3x+5)(x-2)},\ x \neq \frac{-5}{3}, 2}$

c) $\dfrac{3\cancel{(x+2)}}{x^2} \times \dfrac{\cancel{x}}{\cancel{x}\cancel{(x+2)}}$

$\boxed{\dfrac{3}{x^2},\ x \neq -2, 0}$

d) $\dfrac{xy \cdot \frac{2}{x} + \frac{3}{xy} \cdot xy}{xy \cdot \frac{2}{xy} + \frac{3}{y} \cdot xy} \rightarrow \dfrac{2y+3}{2+3x}$

$\Rightarrow \boxed{\dfrac{2y+3}{3x+2} \quad x \neq \frac{-2}{3}, 0 \quad y \neq 0}$

2 a) $\dfrac{3}{x-2} + \dfrac{6}{(x-2)(x-3)} = \dfrac{4}{x-3}$

$3(x-3)+6 = 4(x-2)$

$3x-9+6 = 4x-8$

$-x = -5$

$\boxed{x=5}$

x=5 is a solution

b) $\dfrac{x+6}{(x+2)(x-2)} = \dfrac{2}{x-2} + \dfrac{x}{x+2}$

$x+6 = 2(x+2) + x(x-2)$

$x+6 = 2x+4+x^2-2x$

$0 = x^2-x-2$

$0 = (x-2)(x+1)$

$x=2 \mid \boxed{x=-1}$

x=-1 is a solution

② c) $\left(\sqrt{3x+15}\right)^2 = \left(1+\sqrt{18+x}\right)^2$

$3x+15 = 1 + 2\sqrt{18+x} + 18 + x$

$2x-4 = 2\sqrt{18+x}$

$2(x-2) = 2\sqrt{18+x}$

$(x-2)^2 = \left(\sqrt{18+x}\right)^2$

$x^2-4x+4 = 18+x$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$\boxed{x=7}$ or $x=-2$

$x=7$ is a solution
$x=-2$ is an extraneous root

d) $\left|\dfrac{x+2}{x+1}\right| \leq 2$

Case 1 $x+1>0$		Case 2 $x+1<0$	
$\dfrac{x+2}{x+1} \leq 2$	$\dfrac{x+2}{x+1} \geq -2$	$\dfrac{x+2}{x+1} \leq 2$	$\dfrac{x+2}{x+1} \geq -2$
$x+2 \leq 2x+2$	$x+2 \geq -2x-2$	$x+2 \geq 2x+2$	$x+2 \leq -2x-2$
$-x \leq 0$	$3x \geq -4$	$-x \geq 0$	$3x \leq -4$
$\boxed{x \geq 0}$	$x \geq -\frac{4}{3}$	$x \leq 0$	$\boxed{x \leq -\frac{4}{3}}$

e) $|2x-7| \geq 15$

① $2x-7 \geq 15$
$2x \geq 22$
$\boxed{x \geq 11}$

② $2x-7 \leq -15$
$2x \leq -8$
$\boxed{x \leq -4}$

f) $12 > |x-5| > -8$

① $12 > x-5 > -8$
$17 > x > -3$
$\boxed{-3 < x < 17}$

② $-12 < x-5 < 8$
$\boxed{-7 < x < 13}$

③ a) $f(x) = \dfrac{x^2+5x}{x^2+10x+25} = \dfrac{x\cancel{(x+5)}}{\cancel{(x+5)}(x+5)} = \dfrac{x}{x+5}$

① roots: $x=0$

② V.A $x=-5$

③ H.A. $y=1$

④ Holes! None

⑤ y int! $y=0$

③ b) $f(x) = \dfrac{x^2+9x+8}{x^3+3x^2-10x} = \dfrac{(x+1)(x+8)}{x(x+5)(x-2)}$

① Roots:	② V.A.	③ H.A.	④ Holes:	y int:
$x=-8,-1$	$x=-5,0,2$	$y=0$	None	None

c) $f(x) = \dfrac{x^2+8x+12}{x+5} = \dfrac{(x+6)(x+2)}{x+5}$

① Roots	② V.A	③ O.A.	④ Holes:	⑤ y int
$x=-6,-2$	$x=-5$	(see division below)	None	$y=12/5$

$$\begin{array}{r|l} & x+3 \\ x+5 & x^2+8x+12 \\ & -(x^2+5x)\downarrow \\ \hline & 3x+12 \\ & -(3x+15) \\ \hline & -3R \end{array}$$

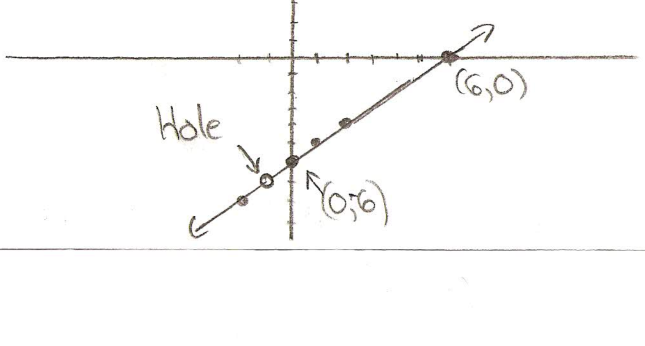
$y=x+3$

d) $f(x) = \dfrac{x^2-2x-3}{x+1} = \dfrac{(x-3)\cancel{(x+1)}}{\cancel{x+1}} = x-3$

① Roots:	② V.A.	③ O.A.	④ Holes:	⑤ y int
$x=3$	None	$y=x-3$	$x=-1$	$y=-3$

④ a) $f(x) = \dfrac{x^2-5x-6}{x+1} = \dfrac{(x-6)\cancel{(x+1)}}{\cancel{(x+1)}} = x-6$

① Root	② V.A.	③ O.A.	④ Hole!	⑤ y int
$x=6$	None	$y=x-6$	$x=-1$	$y=-6$



*④ b) $f(x) = \dfrac{x^2-2x-3}{x^2+6x+8} = \dfrac{(x-3)(x+1)}{(x+2)(x+4)}$

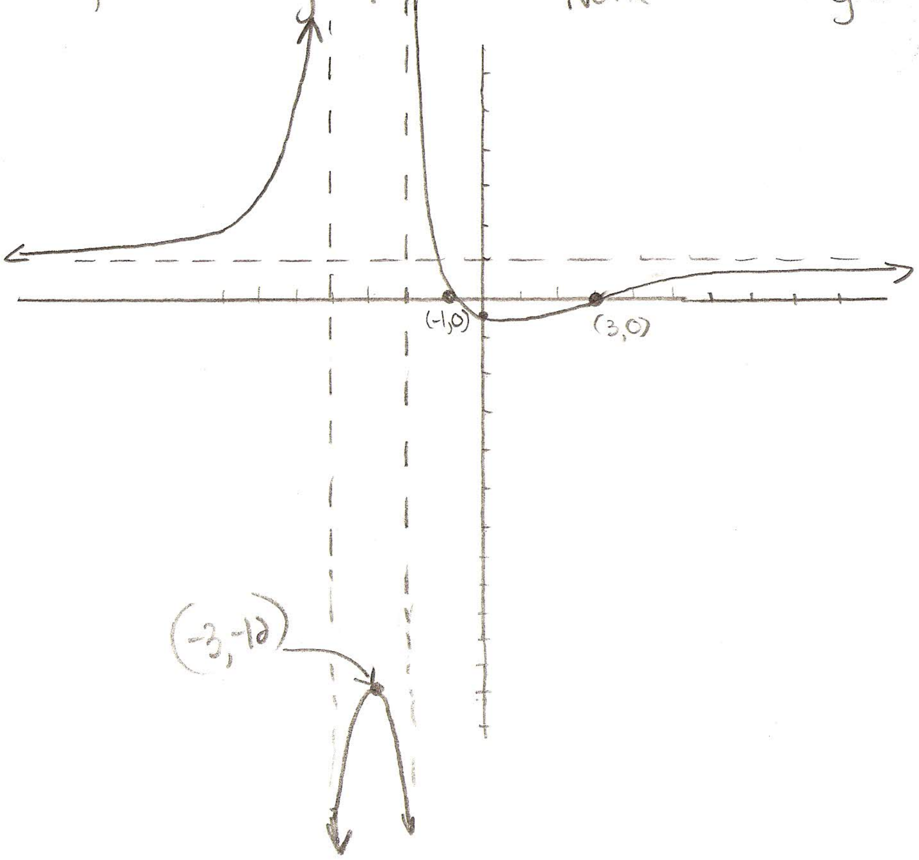
① Roots $x = 3, -1$ ② V.A. $x = -4, -2$ ③ H.A $y = 1$ ④ Holes: None ⑤ y int $y = -3/8$

$\lim_{x \to -4^-} f(x) = +\infty$

$\lim_{x \to -4^+} f(x) = -\infty$

$\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = +\infty$



c) $f(x) = \dfrac{x^2-4}{x^2-9} = \dfrac{(x+2)(x-2)}{(x+3)(x-3)}$

① Roots $x = \pm 2$ ② V.A $x = \pm 3$ ③ H.A. $y = 1$ ④ Holes None ⑤ y int $y = 4/9$

$\lim_{x \to -3^-} f(x) = +\infty$

$\lim_{x \to -3^+} f(x) = -\infty$

$\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 3^+} f(x) = +\infty$