Integration Exam Review:

① a) $f(x) = x^3 - 3x^2 + 5$

$F(x) = \frac{x^4}{4} - \frac{3x^3}{3} + 5x + C$

$\boxed{F(x) = \frac{x^4}{4} - x^3 + 5x + C}$

b) $f(x) = 7x^3 + 9x^2 + 8x - 1$

$F(x) = \frac{7x^4}{4} + \frac{9x^3}{3} + \frac{8x^2}{2} - x + C$

$\boxed{F(x) = \frac{7x^4}{4} + 3x^3 + 4x^2 - x + C}$

c) $\int (x^{5/6} - 3x^{9/2} + x^{-6} - 3x^{-1/2})\,dx$

$= \frac{x^{11/6}}{\frac{11}{6}} - \frac{3x^{11/2}}{\frac{11}{2}} + \frac{x^{-5}}{-5} - \frac{3x^{1/2}}{1/2} + C$

$\boxed{= \frac{6}{11}x^{11/6} - \frac{6}{11}x^{11/2} - \frac{1}{5x^5} - 6x^{1/2} + C}$

d) $f'(x) = \sec 3x \tan 3x$

$\boxed{f(x) = \frac{1}{3}\sec 3x + C}$

e) $f(x) = e^{9x}$

$\boxed{F(x) = \frac{e^{9x}}{9} + C}$

f) $\int (x^3 + 9x^{-5} + \frac{2}{x} + 7e^{-2x})\,dx$

$= \frac{x^4}{4} + \frac{9x^{-4}}{-4} + 2\ln|x| + \frac{7e^{-2x}}{-2} + C$

$\boxed{= \frac{x^4}{4} - \frac{9}{4x^4} + 2\ln x - \frac{7}{2}e^{-2x} + C}$

② a) On back

b) $\int_{-1}^{2} (x^3 - 2x^2 + 6)\,dx = \left.\frac{x^4}{4} - \frac{2x^3}{3} + 6x\right]_{-1}^{2}$

$= \frac{(2)^4}{4} - \frac{2(2)^3}{3} + 6(2) - \left[\frac{(-1)^4}{4} - \frac{2(-1)^3}{3} + 6(-1)\right]$

$= 4 - \frac{16}{3} + 12 - \left[\frac{1}{4} + \frac{2}{3} - 6\right]$

$= \frac{32}{3} + \frac{61}{12}$

$= \frac{189}{12} = \boxed{\frac{63}{4}}$

② a) $f(x) = x^3 - 2x^2 + 6$ from $x = -1$ to $x = 2$

① $\Delta x = \frac{b-a}{n}$
$= \frac{3}{n}$

② $f(x_i^*) = f(a + i\Delta x)$
$= f\left(-1 + \frac{3i}{n}\right)$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1 + \frac{3i}{n}\right)\frac{3}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-1 + \frac{3i}{n}\right)^3 - 2\left(-1 + \frac{3i}{n}\right)^2 + 6\right]\frac{3}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-1 + \frac{9i}{n} - \frac{27i^2}{n^2} + \frac{27i^3}{n^3}\right) - 2\left(1 - \frac{6i}{n} + \frac{9i^2}{n^2}\right) + 6\right]\frac{3}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[-1 + \frac{9i}{n} - \frac{27i^2}{n^2} + \frac{27i^3}{n^3} - 2 + \frac{12i}{n} - \frac{18i^2}{n^2} + 6\right]\frac{3}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{27i^3}{n^3} - \frac{45i^2}{n^2} + \frac{21i}{n} + 3\right]\frac{3}{n}$$

cubic, Quad., Linear, Constant

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{81i^3}{n^4} - \frac{135i^2}{n^3} + \frac{63i}{n^2} + \frac{9}{n}$$

$$A = \lim_{n\to\infty} \frac{81}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{135}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{63}{n^2}\left[\frac{n(n+1)}{2}\right] + \frac{9}{\cancel{n}}\cdot\cancel{n}$$

$$A = \lim_{n\to\infty} \frac{81}{n^4}\left(\frac{n^4 + 2n^3 + n^2}{4}\right) - \frac{135}{n^3}\left(\frac{2n^3 + 3n^2 + n}{6}\right) + \frac{63}{n^2}\left(\frac{n^2 + n}{2}\right) + 9$$

$$A = \lim_{n\to\infty} \frac{81n^4}{4n^4} + \frac{162n^3}{4n^4} + \frac{81n^2}{4n^4} - \frac{270n^3}{6n^3} - \frac{405n^2}{6n^3} - \frac{135n}{6n^3} + \frac{63n^2}{2n^2} + \frac{63n}{2n^2} + 9$$

$$A = \lim_{n\to\infty} \frac{81}{4} + \frac{81}{2n} + \frac{81}{4n^2} - 45 - \frac{135}{2n} - \frac{45}{2n^2} + \frac{63}{2} + \frac{63}{2n} + 9$$

$$A = \lim_{n\to\infty} \frac{81}{4} + 0 + 0 - 45 - 0 - 0 + \frac{63}{2} + 0 + 9 = \boxed{\frac{63}{4}}$$

Integration Exam Review:

③ a) $\int_0^1 (x^2-2x)\,dx = \frac{x^3}{3} - x^2 \Big]_0^1$

$= \frac{1}{3} - 1 - [0-0]$

$= \boxed{-\frac{2}{3}}$

b) $\int_0^{2\pi} \cos x\,dx = \sin x \Big]_0^{2\pi}$

$= \sin(2\pi) - \sin(0)$

$= 0 - 0$

$\boxed{= 0}$

c) $\int_0^1 \frac{1}{1+x^2}\,dx = \tan^{-1} x \Big]_0^1$

$= \tan^{-1}(1) - \tan^{-1}(0)$

$= \frac{\pi}{4} - 0$

$= \boxed{\frac{\pi}{4}}$

d) $\int_2^3 x^5\,dx = \frac{x^6}{6} \Big]_2^3$

$= \frac{729}{6} - \frac{64}{6}$

$\boxed{= \frac{665}{6}}$

e) $\int_\pi^{2\pi} \frac{1}{x}\,dx = \ln|x| \Big]_\pi^{2\pi}$

$= \ln 2\pi - \ln \pi$

$= \boxed{\ln 2}$

f) $\int_1^5 (x^2+2x^3)\,dx = \frac{x^3}{3} + \frac{x^4}{2} \Big]_1^5$

$= \frac{125}{3} + \frac{625}{2} - \left[\frac{1}{3} + \frac{1}{2}\right]$

$= \frac{2125}{6} - \frac{5}{6} = \boxed{\frac{1060}{3}}$

g) $\int_0^{\pi/4} \sec^2 x\,dx = \tan x \Big]_0^{\pi/4}$

$= \tan(\pi/4) - \tan(0)$

$= 1 - 0$

$= \boxed{1}$

h) $\int_{-2}^3 \frac{1}{x^3}\,dx = \boxed{\text{undefined}}$

* $\frac{1}{x^3}$ is discontinuous when $x=0$

i) $\int_0^1 e^{3x}\,dx = \left.\frac{e^{3x}}{3}\right]_0^1$

$= \frac{e^3}{3} - \frac{e^0}{3}$

$\boxed{= \frac{e^3 - 1}{3}}$