Questions from Homework

<u>Velocity</u>

④ $h = 24.5t - 4.9t^2$

$v = 24.5 - 9.8t$

b) Let $v = 0$

$0 = 24.5 - 9.8t$

$9.8t = 24.5$

$t = 2.5s$

c) Find $h(2.5)$

$h = 24.5(2.5) - 4.9(2.5)^2$

$h = 30.6m$

d) Let $h = 0$

$0 = 24.5t - 4.9t^2$

$0 = t(24.5 - 4.9t)$

$t = 0$ | $24.5 - 4.9t = 0$

$24.5 = 4.9t$

$\boxed{5 = t}$

④ d) $s = \dfrac{5t}{1+t}$

$v = \dfrac{(1+t)(5) - 5t(1)}{(1+t)^2}$

$v = \dfrac{5+5t-5t}{(1+t)^2} = \dfrac{5}{(1+t)^2}$

$a = \dfrac{(1+t)^2(0) - 5(2)(1+t)(1)}{[(1+t)^2]^2}$

$a = \dfrac{-10(1+t)}{(1+t)^4} = \dfrac{-10}{(1+t)^3}$

$a(4) = \dfrac{-10}{(1+4)^3} = \dfrac{-10}{125} = \boxed{\dfrac{-2}{25} \ m/s^2}$

③ d) $s = \sqrt{t^2+t} = (t^2+t)^{1/2}$

$v = \frac{1}{2}(t^2+t)^{-1/2}(2t+1)$

$v = \frac{2t+1}{2(t^2+t)^{1/2}}$

$a = \frac{2(t^2+t)^{1/2}(2) - (2t+1)(1)(t^2+t)^{-1/2}(2t+1)}{[2(t^2+t)^{1/2}]^2}$

$a = \frac{4(t^2+t)^{1/2} - (2t+1)^2(t^2+t)^{-1/2}}{4(t^2+t)}$

FOIL

$a = \frac{(t^2+t)^{-1/2}[4(t^2+t) - (2t+1)^2]}{4(t^2+t)}$

$a = \frac{4t^2 + 4t - 4t^2 - 4t - 1}{4(t^2+t)^{3/2}}$

$a = \frac{-1}{4(t^2+t)^{3/2}}$

⑧ $s = t^3 - 15t^2 + 63t$

$v = 3t^2 - 30t + 63$

a) Let $v = 0$

$0 = 3t^2 - 30t + 63$

$0 = 3(t^2 - 10t + 21)$

$0 = 3(t-7)(t-3)$

$t - 7 = 0$ | $t - 3 = 0$

$t = 7s$ | $t = 3s$

b) $v > 0$

$3t^2 - 30t + 63 > 0$

$3(t-7)(t-3) > 0$

① $t - 7 > 0$ | $t - 3 > 0$

$\boxed{t > 7}$ | $t > 3$

② $t - 7 < 0$ | $t - 3 < 0$

$t < 7$ | $\boxed{t < 3}$

Related Rates

In a related rates problem, we are given the rate of change of one quantity and we are to find the rate of change of a related quantity. To do this, we find an equation that relates the two quantities and use the *Chain Rule* to differentiate both sides of the equation *with respect to time.*

Differentiate with respect to time

If $xy^2 = 12$ and $\underline{\dfrac{dy}{dt}} = 6$, Find $\boxed{\underline{\dfrac{dx}{dt}}}$ when $y = 2$

$$xy^2 = 12$$
$$x(2)^2 = 12$$
$$\underline{4x = 12}$$
$$\boxed{x = 3}$$

$$xy^2 = 12$$

$$x \cdot 2y\frac{dy}{dt} + \boxed{\frac{dx}{dt}}y^2 = 0$$

$$(3)2(2)(6) + \left(\frac{dx}{dt}\right)(2)^2 = 0$$

$$72 + 4\frac{dx}{dt} = 0$$

$$4\frac{dx}{dt} = -72$$

$$\boxed{\frac{dx}{dt} = -18}$$

If $x^3 + y^3 = 9$ and $\dfrac{dx}{dt} = 4$, Find $\dfrac{dy}{dt}$ when $x = 2$

$x^3 + y^3 = 9$

$(2)^3 + y^3 = 9$

$8 + y^3 = 9$

$y^3 = 1$

$\boxed{y = 1}$

$3x^2 \dfrac{dx}{dt} + 3y^2 \dfrac{dy}{dt} = 0$

$3(2)^2(4) + 3(1)^2 \dfrac{dy}{dt} = 0$

$48 + 3\dfrac{dy}{dt} = 0$

$3\dfrac{dy}{dt} = -48$

$\boxed{\dfrac{dy}{dt} = -16}$