Specialized Factoring Techniques

- **Common Factors**
- **The Sum and Difference of Cubes**
- **Grouping to Find a Common Factor**
- **Quartic Expressions Factored as Trinomials**
- **Grouping to get the Difference of Squares**

Let's Start with a quick refresher!

Common Factor

$$12x^7 y^8 + 24x^9 y^4$$

Simple Trinomial

$$x^2 - 5x + 6$$

Trinomial Decomposition

$$4x^2 + 5x - 6$$

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

$$81x^2 - 49b^2$$

Common Factor

$$x^3 - x^2 - 12x$$

Difference of Cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$x^3 - 8$$

$$27x^3 - 64$$

Sum of Cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$x^3 + 27$$

$$64x^3 + 125$$

Grouping to Find a Common Factor

A common factor can sometimes be found for specific groups of terms in a polynomial expression. The expression is written in the necessary order and each group of terms is then factored, leaving a common factor in brackets, which in turn is factored.

$$x^3 - 2x^2 - 16x + 32$$

$$x^3 + 4x^2 - 4x - 16$$

$$8x^5 - 40x^4 + 32x^3 - x^2 + 5x - 4$$

Questions From Homework

② d) $(x^5 - 5x^4)(10x^3 + 50x^2)(9x - 45)$

$x^4(x-5) - 10x^2(x-5) + 9(x-5)$

$(x-5)(x^4 - 10x^2 + 9)$

$(x-5)(x^2-1)(x^2-9)$

$(x-5)(x-1)(x+1)(x-3)(x+3)$

Quartic Expressions Factored as Trinomials

$$x^4 - 5x^2 + 4$$

$$(x^2 - 1)(x^2 - 4)$$

$$(x+1)(x-1)(x-2)(x+2)$$

$$4x^4 - 37x^2 + 9$$

$-36 \times -1 = 36$

$-36 + -1 = -37$

$$(4x^4 - x^2)(36x^2 + 9)$$

$$x^2(4x^2 - 1) - 9(4x^2 - 1)$$

$$(4x^2 - 1)(x^2 - 9)$$

$$(2x-1)(2x+1)(x-3)(x+3)$$

Grouping to Get the Difference of Squares

If a polynomial expression can be grouped in the form $(x+m)^2-n^2$, then it can be factored as the difference of squares.

$$\left(x^2 - 6x + 9\right) - b^2$$

$$(x-3)(x-3) - b^2$$

$$(x-3)^2 - b^2$$

$$\left((x-3) - b\right)\left((x-3) + b\right)$$

$$(x-3-b)(x-3+b)$$

$$x^4 + 8x^2 - a^2 + 16$$

$$\left(x^4 + 8x^2 + 16\right) - a^2$$

$$\left(x^2 + 4\right)\left(x^2 + 4\right) - a^2$$

$$\left(x^2 + 4\right)^2 - a^2$$

$$\left((x^2+4) - a\right)\left((x^2+4) + a\right)$$

$$(x^2+4-a)(x^2+4+a)$$

$$16x^2 + 40x - a^2 + 25$$

$$\left(16x^2 + 40x + 25\right) - a^2$$

$$(4x+5)(4x+5) - a^2$$

$$(4x+5)^2 - a^2$$

$$\left((4x+5) + a\right)\left((4x+5) - a\right)$$

$$(4x+5+a)(4x+5-a)$$

Find the Missing Term First

$$\sqrt{9} = 3 \cdot 2 = 6$$

$$x^4 + \underline{5x^2} + 9$$

$$\left(x^4 + 6x^2 + 9\right) - x^2$$

$$\boxed{\left(x^2 + 3\right)^2} - \boxed{x^2}$$

$$\left(\left(x^2 + 3\right) - x\right)\left(\left(x^2 + 3\right) + x\right)$$

$$\left(x^2 - x + 3\right)\left(x^2 + x + 3\right)$$

$$\sqrt{1} = 1 \cdot 2 = 2$$

$$x^4 \underline{\underline{-6x^2}} + 1$$

$$\left(x^4 - 2x^2 + 1\right) - 4x^2$$

$$\boxed{\left(x^2 - 1\right)^2} - \boxed{4x^2}$$

$$\left(\left(x^2 - 1\right) - 2x\right)\left(\left(x^2 - 1\right) + 2x\right)$$

$$\left(x^2 - 2x - 1\right)\left(x^2 + 2x - 1\right)$$