Warm Up

Differentiate each of the following:

1. $f(x) = 6^{x^3} + \ln\left(\tan^{-1} 2x^4\right)$

$$f'(x) = 6^{x^3}(\ln 6)(3x^2) + \left(\frac{1}{\tan^{-1} 2x^4}\right)\left(\frac{8x^3}{1+(2x^4)^2}\right)$$

2. $y = (8x-1)^{\sqrt{x}}$

$$\ln y = \ln(8x-1)^{\sqrt{x}}$$

$$\ln y = \left[x^{1/2}\right]\left[\ln(8x-1)\right]$$

$$y\,\frac{y'}{y} = \left[x^{1/2}\left(\frac{8}{8x-1}\right) + \frac{1}{2}x^{-1/2}\left(\ln(8x-1)\right)\right]\left[(8x-1)^{\sqrt{x}}\right]$$

Derivative Rules

Exponential Functions

$d(b^u) = b^u \bullet (\ln b) \bullet du$, where $b \in R$

$d(e^u) = e^u \bullet du$, base is Euler's number

Logarithmic Functions

$d(\log_b u) = \frac{1}{u \ln b} \bullet du$, where $b \in R$

$d(\ln u) = \frac{1}{u} \bullet du$, base is Euler's number

Inverse Trigonometric Functions

$$\frac{d(\sin^{-1} u)}{du} = \frac{1}{\sqrt{1-u^2}} du \qquad \frac{d(\csc^{-1} u)}{du} = \frac{-1}{u\sqrt{u^2-1}} du$$

$$\frac{d(\cos^{-1} u)}{du} = \frac{-1}{\sqrt{1-u^2}} du \qquad \frac{d(\sec^{-1} u)}{du} = \frac{1}{u\sqrt{u^2-1}} du$$

$$\frac{d(\tan^{-1} u)}{du} = \frac{1}{1+u^2} du \qquad \frac{d(\cot^{-1} u)}{du} = \frac{-1}{u^2+1} du$$

Quiz Monday: Derivatives of Transcendental Functions

- Inverse Trigonometric Functions
- Exponential Functions
- Logarithmic Functions

Practice Test

Solutions

May 06

1 a) $f'(x) = \frac{1}{x^3 \ln 5}(3x^2) + e^{\sin 5x}(\cos 5x)(5)$

b) $y' = \frac{-1}{\sqrt{1-(2/x)^2}}(x^{-1/2}) - \frac{1}{\ln x^2}\left(\frac{1}{x^2}\right)(2x)$

c) $h'(t) = \frac{5^{\tan t}\ln 5\,(\sec^2 t)\ln(3t^4+5) - 5^{\tan t}\left(\frac{1}{3t^4+5}(12t^3)\right)}{[\ln(3t^4+5)]^2}$

d) $\ln y = x \ln(5-2x^2)$

$\frac{1}{y}y' = \left[\ln(5-2x^2) + x\left(\frac{1}{5-2x^2}\right)(-4x)\right]y$

$y' = \left(\ln(5-2x^2) - \frac{4x^2}{5-2}\right)(5-2x^2)^x$

e) $y' = \frac{1}{1+[\ln^3(x^5-1)]^2}\left[3[\ln(x^5-1)]^2\left(\frac{1}{x^5-1}\right)(5x^4)\right]$

f) $g'(x) = 4^{5x}\ln 4\,(5)\,e^{\sin^{-1}\sqrt{x}} + 4^{5x}e^{\sin^{-1}\sqrt{x}}\left(\frac{1}{\sqrt{1-x}}\,\frac{1}{2}x^{-1/2}\right)$

2. $\ln y = 3\ln(x^2-2x) + \ln 8x^5 - \frac{5}{2}\ln(5-x^2) - (x^5+2)$

$\frac{1}{y}y' = \left[3\left(\frac{2x-2}{x^2-2x}\right) + \frac{40x^4}{8x^5} - \frac{5}{2}\left(\frac{1(-2x)}{5-x^2}\right) - 5x^4\right]y$

$y' = \left(\frac{3(2x-2)}{x^2-2x} + \frac{5}{x} + \frac{5x}{5-x^2} - 5x^4\right)\left[\frac{(x^2-2x)^3(8x^5)}{(5-x^2)^{5/2}(e^{x^5+2})}\right]$

3. $e^{3x-y^5}\left(3 - 5y^4\frac{dy}{dx}\right) = 5^{xy^3}\ln 5\left(y^3 + x\left(3y^2\frac{dy}{dx}\right)\right)$

$3e^{3x-y^5} - 5y^4e^{3x-y^5}\frac{dy}{dx} = 5^{xy^3}\ln 5\,y^3 + 3xy^2 5^{xy^3}\ln 5\frac{dy}{dx}$

$\frac{3e^{3x-y^5} - 5^{xy^3}\ln 5\,y^3}{3xy^2 5^{xy^3}\ln 5 + 5y^4e^{3x-y^5}} = \frac{dy}{dx}$

4. $\frac{dz}{dy} = \frac{1}{\sqrt{1-(y-3)^2}}(1) - 3y^2$ ~~when x = 0~~

$\frac{dy}{dx} = 3e^x + 2x$

$\frac{dz}{dx} = \frac{dz}{dy}\cdot\frac{dy}{dx} = \frac{dz}{dx}$

at $x = 0$ . . . $y = 3 + 0$, $y = 3$

$\frac{dz}{dy} = \frac{1}{\sqrt{1-0^2}} - 27 = -26$ $\frac{dy}{dx} = 3e^0 + 2(0) = 3$

$\boxed{\frac{dz}{dx} = -78}$

5. $f'(x) = 2xe^{2x} + x^2e^{2x}(2)$

$f'(x) = 2xe^{2x}(1+x)$

Critical Values: $x = -1$, $x = 0$

	$2x$	e^{2x}	$1+x$	f'	f
$(-\infty, -1)$	−	+	−	+	Inc
$(-1, 0)$	−	+	+	−	dec
$(0, \infty)$	+	+	+	+	Inc

Local Max: $\left(-1, \frac{1}{e^2}\right)$

Local Min: $(0, 0)$

Attachments

Review of Transcendentals.doc

logs & arcfunctions test 2006.doc