$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1}$$

Warm Up

$$\boxed{\lim_{x\to 0}\frac{x}{\sin x}=1}$$

Evaluate the following limits if they exist.

$$\lim_{x\to 0}\frac{2x^3}{\sin^3 3x}$$

$$\lim_{x\to 0}\left(\frac{x^3}{\sin^3 3x}\right)(2)$$

$$\lim_{x\to 0}\left(\frac{3x}{\sin 3x}\right)^3\frac{(2)}{27}$$

$$\lim_{x\to 0}\left(\frac{3x}{\sin 3x}\right)^3\cdot\lim_{x\to 0}\frac{2}{27}$$

$$=(1)^3\left(\frac{2}{27}\right)$$

$$=\frac{2}{27}$$

$$\lim_{x\to -5^-}\frac{|x+5|}{4x+20}$$

$$\lim_{x\to -5^-}\frac{|x+5|}{4(x+5)}$$

$$\lim_{x\to -5^-}\frac{\cancel{|-5.001+5|}}{\cancel{4(-5.001+5)}}\;-1$$

$$=-\frac{1}{4}$$

Questions from Homework

Identity

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{x}{\sin x}=1$$

9. $\lim\limits_{x\to 0}\dfrac{\sin^3 2x}{\sin^3 3x}$

$$\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^3\left(\frac{3x}{\sin 3x}\right)^3\frac{8\cancel{x^3}}{27\cancel{x^3}}$$

$$\lim_{x\to 0}(1)^3(1)^3\left(\frac{8}{27}\right)=\boxed{\frac{8}{27}}$$

26. $\lim\limits_{x\to 0}\dfrac{2\tan x}{x\sec x}$

$$\lim_{x\to 0}\frac{\frac{2\sin x}{\cos x}}{\frac{x}{\cos x}}$$

$$\lim_{x\to 0}\frac{2\sin x}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{x}$$

$$\lim_{x\to 0}\frac{2\sin x}{x}$$

$$\lim_{x\to 0}(2)\left(\frac{\sin x}{x}\right)=(2)(1)=\boxed{2}$$

31. $\lim\limits_{x\to 0}\dfrac{(\sin^2 x\cos x)(1+\cos x)}{(1-\cos x)(1+\cos x)}$

$$\lim_{x\to 0}\frac{\sin^2 x\cos x(1+\cos x)}{\boxed{(1-\cos^2 x)}}\rightarrow \text{Pyth.}=\sin^2 x$$

$$\lim_{x\to 0}\frac{\cancel{\sin^2 x}\cos x(1+\cos x)}{\cancel{\sin^2 x}}=(1)(2)=\boxed{2}$$

37. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{2x^2+x}$

$$\lim_{x\to 0}\frac{\sin 2x}{x(2x+1)}$$

$$\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)\left(\frac{1}{2x+1}\right)(2)$$

$$\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)\cdot\lim_{x\to 0}\left(\frac{1}{2x+1}\right)\cdot\lim_{x\to 0}2$$

$$=(1)(1)(2)$$

$$=2$$

L'Hopital's Rule:

<u>L'Hôpital's Rule:</u> (if the limit is indeterminant) $\left(= \frac{0}{0} = \frac{\infty}{\infty}\right)$

$$\lim_{x \to n} \frac{f(x)}{g(x)} = \lim_{x \to n} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 3} \frac{2x^2 - x - 15}{x^2 - 9} = \frac{0}{0}$$

$$\lim_{x \to 3} \frac{4x - 1}{2x} = \boxed{\frac{11}{6}}$$

$$\lim_{x \to \pi/2} \frac{1 - \sin x}{\cos x} = \frac{0}{0}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{+\cos x}{+\sin x} = \frac{0}{1} = \boxed{0}$$

$$\lim_{x \to 0} \frac{e^x - 1}{x^3} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x}{3x^2} = \frac{1}{0} = \boxed{\text{DNE}}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\sin x}{2x}$$

$$\lim_{x \to 0} \frac{\cos x}{2} = \boxed{\frac{1}{2}}$$

$$\lim_{x \to \infty} \frac{e^x}{x^2} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{e^x}{2x}$$

$$\lim_{x \to \infty} \frac{e^x}{2} = \frac{\infty}{2} = \boxed{\text{DNE}}$$

Homework

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