Warm Up

Evaluate the following limits: (L'Hopital's Rule)

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}} = \frac{0}{0}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos x + \sin x}{1}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{1} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \boxed{\sqrt{2}}$$

$$\lim_{x\to 0} \frac{\overset{\sin^{-1} x}{Arc\sin x}}{x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1}$$

$$\lim_{x\to 0} \frac{\frac{1}{1}}{1} = \boxed{1}$$

$$\lim_{x\to 0} \frac{e^x - 1}{\sin 2x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{e^x}{\cos 2x(2)}$$

$$\lim_{x\to 0} \frac{1}{(1)(2)} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to\infty} \frac{10x^2}{e^x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{20x}{e^x}$$

$$\lim_{x\to\infty} \frac{20}{e^x}$$

$$\lim_{x\to\infty} \frac{20}{\infty} = \boxed{0}$$

Sigma Notation

A series is the sum of a sequence. We can write a series using sigma notation.

$$\boxed{\sum_{i=1}^{n} t_i = t_1 + t_2 + t_3 + \ldots + t_n}$$

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n

$$1+2+4+\ldots+64 = \sum_{i=1}^{7} 2^{i-1}$$

the terms form a geometric sequence with a = 1, r = 2, $t_n = 1(2)^{n-1}$

$t_n = ar^{n-1}$

$t_n = (1)(2)^{n-1}$

$t_n = 2^{n-1}$

$t_i = 2^{i-1}$

This symbol is read as "the sum of the terms of the sequence given by $t_n = 2^{n-1}$ from n = 1 to n = 7"

Example 1

arithmetic

Express the series 1 + 3 + 5 + 7 + 9 in sigma notation.

$a = 1$

$d = 2$

$t_n = a + (n-1)d$

$t_n = 1 + (n-1)2$

$t_n = 1 + 2n - 2$

$t_n = 2n - 1$

$t_i = 2i - 1$

$$\sum_{i=1}^{5} 2i-1$$

The properties of Sigma Notation that we use in this section are summarized below:

$$\sum_{i=1}^{n} c = c + c + c + \ldots + c = nc$$

$$\sum_{i=1}^{n} ct_i = c\sum_{i=1}^{n} t_i \text{ , } c \text{ is a constant}$$

$$\sum_{i=1}^{n} (t_i + s_i) = \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} s_i$$

Example 2

Use the basic properties of sigma notation to express $\sum_{i=1}^{n}(3i-2)^2$ in terms of monomial summations.

$$= \sum_{i=1}^{n}(9i^2 - 12i + 4)$$

$$= \sum_{i=1}^{n} 9i^2 + \sum_{i=1}^{n}(-12i) + \sum_{i=1}^{n} 4$$

$$= 9\sum_{i=1}^{n} i^2 - 12\sum_{i=1}^{n} i + 4n$$

The following sigma formulas will be extremely useful in the next few days when we are faced with the challenge of calculating the area under a curve.

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + 4 + \ldots . n = \frac{n(n+1)}{2}$$ (Linear)

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + \ldots . n^2 = \frac{n(n+1)(2n+1)}{6}$$ (Quad)

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + 4^3 + \ldots n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

(Cubic)

Example 3

Evaluate: $\sum_{i=1}^{n} \left(3i^2 - 2i\right)$

$$= \sum_{i=1}^{n} 3i^2 + \sum_{i=1}^{n} (-2i)$$

$$= 3\boxed{\sum_{i=1}^{n} i^2} - 2\boxed{\sum_{i=1}^{n} i}$$

$$= \cancel{3}^{1}\left(\frac{n(n+1)(2n+1)}{\cancel{6}\,2}\right) - 2\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{n(n+1)(2n+1) - 2n(n+1)}{2}$$ ← Factor

$$= \frac{n(n+1)\left[(2n+1) - 2\right]}{2}$$

$$\boxed{= \frac{n(n+1)(2n-1)}{2}}$$

Example 4

$$\sum_{i=11}^{20}(2i^2-3i) = \sum_{i=1}^{20}(2i^2-3i) - \sum_{i=1}^{10}(2i^2-3i)$$

$$= 2\boxed{\sum_{i=1}^{20} i^2} - 3\boxed{\sum_{i=1}^{20} i} - \left[2\boxed{\sum_{i=1}^{10} i^2} - 3\sum_{i=1}^{10} i\right]$$

$$= \cancel{2}^{1}\left(\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right) - 3\left(\frac{n(n+1)}{2}\right) - \left[\cancel{2}^{1}\left(\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right) - 3\left(\frac{n(n+1)}{2}\right)\right]$$

$$= \frac{(20)(21)(41)}{3} - 3\left(\frac{20(21)}{2}\right) - \left[\frac{10(11)(21)}{3} - 3\left(\frac{10(11)}{2}\right)\right]$$

$$= 5740 - 630 - [770 - 165]$$

$$= 5110 - 605$$

$$= 4505$$

Homework

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