CIRCLES with CENTER at (h, k)

The equation of a circle centered at any point, (**h, k**) with a radius of **r**, is given by:

$$(x - \mathbf{h})^2 + (y - \mathbf{k})^2 = \mathbf{r}^2$$

This equation of a circle is also said to be expressed in **standard form**.

*** It should be readily seen that this equation reduces to $\mathbf{x}^2 + \mathbf{y}^2 = \mathbf{r}^2$ if the point (**0**, **0**) is substituted for (**h**, **k**).

Example 1

Determine the center and radius for each circle defined by the following equations:

A: $(x-4)^2+(y+5)^2=36$ B: $(x+9)^2+(y-3)^2=14$

Solution

A: Center (4, -5); $r=\sqrt{36}=6$ B: Center (-9, 3); $r=\sqrt{14}$

C: $(x+4)^2+(y-3)^2=20$

Center: $(-4,3)$ $r=\sqrt{20}$ $r=\sqrt{4\times5}$ $r=2\sqrt{5}$

Example 2

Give the equation of the circle in standard form with a radius of 4 units and a center at (-3, 6).

$(x+3)^2 + (y-6)^2 = (4)^2$

Solution

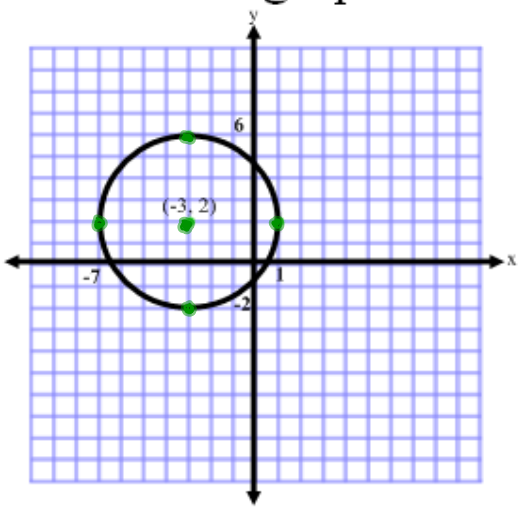
$\mathbf{(x - h)^2 + (y - k)^2 = r^2}$

$(x - -3)^2 + (y - 6)^2 = (4)^2$

$\mathbf{(x + 3)^2 + (y - 6)^2 = 16}$

Example 3

Given the equation, $(x+3)^2 + (y-2)^2 = 16$, find the center and radius of the circle and sketch the graph. What is the Domain? Range?



Solution

Center $(-3, 2)$; $r = \sqrt{16}$

$= 4$

Domain: $\{x \mid -7 \leq x \leq 1,\ x \in R\}$

Range: $\{y \mid -2 \leq y \leq 6,\ y \in R\}$

**For the circle with a defining equation of $(x - \mathbf{h})^2 + (y - \mathbf{k})^2 = \mathbf{r}^2$, you can see that:

Domain: $\{x \mid -7 \leq x \leq 1,\ x \in R\}$

Range: $\{y \mid -2 \leq y \leq 6,\ y \in R\}$

$-3-4$ $-3+4$ $2-4$ $2+4$

$(-3, 2)$

$h = -3$

$k = 2$

$r = 4$

where the __'s in the domain are filled with $h \pm r$, and the __'s in the range are filled with $k \pm r$.

Homework

1-4

③ $x^2+y^2+8x-4y+3=0$

a) $M(-3,-2)$

x, y

$(-3)^2+(-2)^2+8(-3)-4(-2)+3=0$

$9+4-24+8+3=0$

$0=0$

Since LHS = RHS the $(-3,-2)$ is on the circle