

SOLUTIONS => EQUATIONS OF A CIRCLE # 2

1. a)  $x^2 + y^2 = 49$

Center (0, 0)

$$r = \sqrt{49} = 7$$

b)  $x^2 + (y-3)^2 = 16$

Center (0, 3)

$$r = \sqrt{16} = 4$$

c)  $(x-3)^2 + y^2 = 49$

Center (3, 0)

$$r = \sqrt{49} = 7$$

d)  $x^2 + (y-2)^2 = 64$

Center (0, 2)

$$r = \sqrt{64} = 8$$

e)  $(x-5)^2 + (y+3)^2 - 16 = 0$   
 $(x-5)^2 + (y+3)^2 = 16$

Center (5, -3)

$$r = \sqrt{16} = 4$$

f)  $(x+4)^2 + (y-2)^2 = 100$

Center (-4, 2)

$$r = \sqrt{100} = 10$$

$$2a) C(0,0); r=2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-0)^2 = (2)^2$$

$$* x^2 + y^2 = 4$$

$$b) C(0,-2); r=5$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-(-2))^2 = (5)^2$$

$$x^2 + (y+2)^2 = 25$$

$$c) C(4,0); r=3$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-4)^2 + (y-0)^2 = (3)^2$$

$$(x-4)^2 + y^2 = 9$$

$$d) C(h,k); r=p$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-h)^2 + (y-k)^2 = (p)^2$$

$$(x-h)^2 + (y-k)^2 = p^2$$

$$3. x^2 + y^2 + 8x - 4y + 3 = 0$$

$$a) M(-3, -2)$$

If  $x = -3$  and  $y = -2$ :

$$\begin{array}{l} \text{L.S.} \qquad \qquad \qquad \text{R.S.} \\ x^2 + y^2 + 8x - 4y + 3 \qquad \qquad \qquad 0 \\ = (-3)^2 + (-2)^2 + 8(-3) - 4(-2) + 3 \\ = 9 + 4 - 24 + 8 + 3 \\ = 0 \end{array}$$

Since  $L.S = R.S$ ,  $M(-3, -2)$  is located on the circle.

$$b) N(-2, 1)$$

If  $x = -2$  and  $y = 1$

$$\begin{array}{l} \text{L.S.} \qquad \qquad \qquad \text{R.S.} \\ x^2 + y^2 + 8x - 4y + 3 \qquad \qquad \qquad 0 \\ = (-2)^2 + (1)^2 + 8(-2) - 4(1) + 3 \\ = 4 + 1 - 16 - 4 + 3 \\ = -12 \end{array}$$

Since  $L.S \neq R.S$ ,  $N(-2, 1)$  is not located on the circle.

c)  $S(1, 2)$

If  $x=1$  and  $y=2$ :

<u>L.S</u>	<u>R.S</u>
$x^2 + y^2 + 8x - 4y + 3$	0
$= (1)^2 + (2)^2 + 8(1) - 4(2) + 3$	
$= 1 + 4 + 8 - 8 + 3$	
$= 8$	

Since  $L.S \neq R.S$ ,  $S(1, 2)$  is not located on the circle.

4.  $x^2 + y^2 - 4x + my - 18 = 0$

If  $A(7, 3) \Rightarrow x=7$  and  $y=3$

$$(7)^2 + (3)^2 - 4(7) + m(3) - 18 = 0$$

$$\underline{49} + \underline{9} - \underline{28} + 3m - \underline{18} = 0$$

$$30 + 3m - 18 = 0$$

$$3m + 12 = 0$$

$$\frac{3m}{3} = \frac{-12}{3}$$

$$m = -4$$

⑤ a) Given:  $x^2 + y^2 = r^2$   
 Center  $(0,0)$   $(4)^2 + (0)^2 = r^2$   
 x int:  $(4,0)$   $16 + 0 = r^2$   
 $x$   $y$   $16 = r^2$   
 $4 = r$

Equation:  $x^2 + y^2 = r^2$   
 $x^2 + y^2 = 16$

b) Given:  $(2,-7)$  and  $(4,3)$

①  $M = \left[ \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right]$       ②  $(x-h)^2 + (y-k)^2 = r^2$   
 $(2-3)^2 + (-7+2)^2 = r^2$   
 $(-1)^2 + (-5)^2 = r^2$   
 $1 + 25 = r^2$   
 $26 = r^2$   
 $\sqrt{26} = r$

$= \left[ \frac{2+4}{2}, \frac{-7+3}{2} \right]$   
 $= (3, -2)$  Center  
 $h, k$

③ Equation:  
 $(x-h)^2 + (y-k)^2 = r^2$   
 $(x-3)^2 + (y+2)^2 = 26$

5 a) C(0,0) ; x-int=4  
(4,0)

To find  $r^2$ :

$$\begin{aligned}x^2 + y^2 &= r^2 \\(4)^2 + (0)^2 &= r^2 \\16 + 0 &= r^2 \\16 &= r^2\end{aligned}$$

EQUATION:

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 16\end{aligned}$$

b)  $(2, -7)$  and  $(4, 3)$

To find the center:

$$M = \left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

$$\begin{aligned} M &= \left( \frac{4+2}{2}, \frac{3-7}{2} \right) \\ &= \left( \frac{6}{2}, \frac{-4}{2} \right) \\ &= (3, -2) \\ &\quad \begin{matrix} h & k \end{matrix} \end{aligned}$$

Using:  $C(h, k)$  and  $(x, y)$

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 \\ (4-3)^2 + (3-2)^2 &= r^2 \\ (1)^2 + (1)^2 &= r^2 \\ 1 + 1 &= r^2 \\ 2 &= r^2 \\ \sqrt{2} &= r \end{aligned}$$

$$C(h, k) \quad r = \sqrt{26}$$

$$\begin{aligned} \text{EQUATION: } (x-h)^2 + (y-k)^2 &= r^2 \\ (x-3)^2 + (y-2)^2 &= (\sqrt{26})^2 \\ (x-3)^2 + (y+2)^2 &= 26 \end{aligned}$$

$$C \begin{matrix} h & k \\ (2, 3) \end{matrix} \quad \begin{matrix} x & y \\ (4, -1) \end{matrix}$$

$$\begin{aligned} c) \quad (x-h)^2 + (y-k)^2 &= r^2 \\ (4-2)^2 + (-1-3)^2 &= r^2 \\ (2)^2 + (-4)^2 &= r^2 \\ 4 + 16 &= r^2 \end{aligned}$$

$$\begin{aligned} \sqrt{20} &= \sqrt{4 \times 5} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} 20 &= r^2 \\ \sqrt{20} &= r \\ 2\sqrt{5} &= r \end{aligned}$$

$$\begin{aligned} C \begin{matrix} h & k \\ (2, 3) \end{matrix} \\ r = \sqrt{20} \\ \text{or} \\ 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{EQUATION: } (x-h)^2 + (y-k)^2 &= r^2 \\ (x-2)^2 + (y-3)^2 &= (\sqrt{20})^2 \\ (x-2)^2 + (y-3)^2 &= 20 \end{aligned}$$



$$6 \text{ a) } C \begin{matrix} h & k \\ (4, & -3) \end{matrix}$$

$$A \begin{matrix} x & y \\ (6, & 1) \end{matrix}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(6-4)^2 + (1-(-3))^2 = r^2$$

$$(2)^2 + (4)^2 = r^2$$

$$4 + 16 = r^2$$

$$20 = r^2$$

$$\sqrt{20} = r$$

$$2\sqrt{5} = r$$

$$\sqrt{20} = \sqrt{4 \times 5} \\ = 2\sqrt{5}$$

$$C \begin{matrix} h & k \\ (4, & -3) \end{matrix} \\ r = \sqrt{20} \\ \text{or} \\ 2\sqrt{5}$$

EQUATION:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-4)^2 + (y-(-3))^2 = (\sqrt{20})^2$$

$$(x-4)^2 + (y+3)^2 = 20$$

b)  $C(-3, -4)$ ; Passes through point of intersection of  $2x+y=3$  &  $5x-3y=2$

To find the point of intersection, remember there are 3 methods (substitution, elimination, and graphing)

I will use substitution:

$$2x+y=3 \text{ ①}$$

$$5x-3y=2 \text{ ②}$$

$$\text{① } 2x+y=3$$

$$y = -2x+3 \text{ sub in ②}$$

$$\text{② } 5x-3y=2$$

$$5x-3(-2x+3)=2$$

$$5x+6x-9=2$$

$$11x = 2+9$$

$$\frac{11x}{11} = \frac{11}{11}$$

$$x = 1 \text{ sub in ①}$$

$$\text{① } 2x+y=3$$

$$2(1)+y=3$$

$$2+y=3$$

$$y=3-2$$

$$y=1$$

Point  $(1, 1)$

6b) C  $\begin{matrix} h & k \\ (-3, -4) \end{matrix}$  pt  $\begin{matrix} x & y \\ (1, 1) \end{matrix}$

To find  $r^2$  ∴  $(x-h)^2 + (y-k)^2 = r^2$   
 $(1+3)^2 + (1+4)^2 = r^2$   
 $(4)^2 + (5)^2 = r^2$   
 $16 + 25 = r^2$   
 $41 = r^2$

EQUATION ∴  $(x-h)^2 + (y-k)^2 = r^2$   
 $(x+3)^2 + (y+4)^2 = 41$