Coordinates of the Centre and Radius

The **general form** of a circle is given as:

$$\mathbf{Ax^2 + By^2 + Cx + Dy + E = 0}$$

$$2x^2 + 2y^2 + 3x + 5y + 10 = 0$$

When a circle is given in **general form** we must convert it into **standard form** to obtain the coordinates of the center and the length of the radius.

Remember that in **standard form**, we can state the center and radius of our circle easily.

To convert the equation of a circle from **general form** into **standard form** we make use of a strategy called *completing the square*.

Completing the Square

*Note: When ***completing the square***, the coefficients of the squared terms must be **1**.

If you are given the equation of a circle in **general form**, such as $\mathbf{x^2 + y^2 + 6x - 10y - 12 = 0}$, you can use the following steps to convert it into **standard form**.

$$(x^2+6x+\underline{9})(+y^2-10y+\underline{25}) = 12+9+25$$

$$(x+3)^2+(y-5)^2 = 46$$

Center: $(-3,5)$

Radius $= \sqrt{46}$

$$\mathbf{x^2 + y^2 + 6x - 10y - 12 = 0} \quad \textit{(general form)}$$

Step 1: Separate the **x** and **y** values as shown:

$$\mathbf{x^2 + 6x \qquad + y^2 - 10y \qquad = 12}$$

Step 2: Next, take ½ of the x-value, square it, and add it to the left-hand side of the equation.

To compensate for this amount, you also have to add the same amount to the right-hand side of the equation. Repeat with the y-value.

This results in:

$$x^2 + 6x + \mathbf{9} + y^2 - 10y + \mathbf{25} = 12 + \mathbf{9} + \mathbf{25}$$

$$x^2 + 6x + \mathbf{9} + y^2 - 10y + \mathbf{25} = \mathbf{46}$$

Step 3: The terms $\mathbf{x^2 + 6x + 9}$ can be ***factored*** as: $(x + 3)^2$

The terms $\mathbf{y^2 - 10y + 25}$ can be ***factored*** as: $(y - 5)^2$

Step 4: This leaves you with: $\mathbf{(x + 3)^2 + (y - 5)^2 = 46}$

We can now state that this circle is centered at **(-3, 5)** with a radius of $\sqrt{46}$.

Sometimes, an equation in general form does not result in a circle.

Look at the following examples:

<u>Example 1</u>

General

Find the center and the radius of: $\mathbf{x^2 + y^2 - 4x - 2y + 5 = 0}$

<u>Solution</u>

Step 1: $x^2 - 4x \quad + y^2 - 2y \quad = -5$

Step 2: $x^2 - 4x + \underline{\mathbf{4}} + y^2 - 2y + \underline{\mathbf{1}} = -5 + \mathbf{4} + \mathbf{1}$

Step 3: $(x - 2)^2 + (y - 1)^2 = 0$ (Standard form)

We can know state that the center of the circle is located at **(2, 1)** and that the radius, $\mathbf{r = \sqrt{0} = 0}$

***When the radius is 0, this indicates that the equation represents the point **(2, 1)** only and **<u>not</u>** a circle.

<u>Example 2</u>

Find the center and the radius of: $\frac{2x^2}{2} + \frac{2y^2}{2} - \frac{8x}{2} - \frac{4y}{2} + \frac{12}{2} = \frac{0}{2}$

<u>Solution</u>

The coefficients of the squared terms are not 1!

We have to divide ***each and every term*** by the coefficient of the squared terms (in order to have a circle, both coefficients must be the same).

<u>Extra Step</u>: Divide each term by 2 $\quad \mathbf{x^2 + y^2 - 4x - 2y + 6 = 0}$

Step 1: $\quad x^2 - 4x \quad + y^2 - 2y \quad = -6$

Step 2: $\quad x^2 - 4x + \underline{\mathbf{4}} \quad + y^2 - 2y + \underline{\mathbf{1}} \quad = -6 + \mathbf{4} + \mathbf{1}$

Step 3: $\quad (x - 2)^2 + (y - 1)^2 = \mathbf{-1}$ (Standard)

******We can STOP right here!!! Recall that to find the radius, we take the square root of the value on the right-hand side of the equation. If we try to take the square root of a negative number, we do not get a <u>*real*</u> value for the radius.**

① $x^2+y^2-8x+8y=22$

$$x^2-8x+\underline{16}+y^2+8y+\underline{16}=22+16+16$$

$$(x-4)^2+(y+4)^2=54$$

a) Center $=(4,-4)$

b) $r=\sqrt{54}$

$=\sqrt{9\cdot 6}$

$=3\sqrt{6}$