

ANSWERS => EQUATIONS OF A CIRCLE WORKSHEET #3

1. $x^2 + y^2 - 8x + 8y = 22$

Step 1: $x^2 - 8x + y^2 + 8y = 22$

Step 2: $x^2 - 8x + 16 + y^2 + 8y + 16 = 22 + 16 + 16$

Step 3: $(x - 4)^2 + (y + 4)^2 = 54$

Center $(4, -4)$; $r = \sqrt{54}$
 $r = \sqrt{9 \times 6}$
 $r = 3\sqrt{6}$ units

a) Center => $(4, -4)$

b) Radius => $3\sqrt{6}$

$$2a) x^2 + y^2 - 6y - 3 = 0$$

$$\text{Step 1: } x^2 + y^2 - 6y = 3$$

$$\text{Step 2: } x^2 + y^2 - 6y + 9 = 3 + 9$$

$$\text{Step 3: } (x-0)^2 + (y-3)^2 = 12$$

$$\begin{aligned} \text{Center}(0, 3); \quad r &= \sqrt{12} \\ r &= \sqrt{4 \times 3} \\ r &= 2\sqrt{3} \text{ units} \end{aligned}$$

$$b) x^2 + y^2 - 10x - 1 = 0$$

$$\underline{\text{Step 1}}: x^2 - 10x + y^2 = 1$$

$$\underline{\text{Step 2}}: x^2 - 10x + 25 + y^2 = 1 + 25$$

$$\underline{\text{Step 3}}: (x-5)^2 + (y-0)^2 = 26$$

Center $(5, 0)$; $r = \sqrt{26}$ units.

$$c) x^2 + y^2 - 4x - 2y + 1 = 0$$

$$\underline{\text{Step 1}}: x^2 - 4x + y^2 - 2y = -1$$

$$\underline{\text{Step 2}}: x^2 - 4x + 4 + y^2 - 2y + 1 = -1 + 4 + 1$$

$$\underline{\text{Step 3}}: (x-2)^2 + (y-1)^2 = 4$$

Center $(2, 1)$; $r = \sqrt{4}$
 $r = 2$ units.

$$\textcircled{c) \quad x^2 + y^2 - 4x - 2y + 1 = 0$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = -1 + 4 + 1$$
$$(x-2)^2 + (y-1)^2 = 4$$

Center: $(2, 1)$

$$\text{Radius} = \sqrt{4} = 2$$

$$\text{h) } \frac{2x^2}{2} + \frac{2y^2}{2} + \frac{20x}{2} - \frac{12y}{2} + \frac{18}{2} = \frac{0}{2}$$

$$x^2 + y^2 + 10x - 6y + 9 = 0$$

$$x^2 + 10x + 25 + y^2 - 6y + 9 = -9 + 25 + 9$$
$$(x+5)^2 + (y-3)^2 = 25$$

Center: $(-5, 3)$

$$\text{radius} = \sqrt{25} = 5$$

$$d) x^2 + y^2 + 6x - 8y + 19 = 0$$

$$\underline{\text{Step 1}} \circ x^2 + 6x + y^2 - 8y = -19$$

$$\underline{\text{Step 2}} \circ x^2 + 6x + 9 + y^2 - 8y + 16 = -19 + 9 + 16$$

$$\underline{\text{Step 3}} \circ (x + 3)^2 + (y - 4)^2 = 6$$

Center $(-3, 4)$; $r = \sqrt{6}$ units

$$e) 3x^2 + 3y^2 - 18x + 30y + 100 = 0$$

EXTRA STEP: Divide each term by 3.

$$x^2 + y^2 - 6x + 10y + \frac{100}{3} = 0$$

$$\text{Step 1: } x^2 - 6x + y^2 + 10y = -\frac{100}{3}$$

$$\text{Step 2: } x^2 - 6x + 9 + y^2 + 10y + 25 = -\frac{100}{3} + 9 + 25$$

$$\text{Step 3: } (x-3)^2 + (y+5)^2 = -\frac{100}{3} + \frac{34}{1}$$

$$(x-3)^2 + (y+5)^2 = -\frac{100}{3} + \frac{102}{3}$$

$$(x-3)^2 + (y+5)^2 = \frac{2}{3}$$

Center $(3, -5)$; $r = \sqrt{\frac{2}{3}}$ units.

$$f) x^2 + y^2 + 12x - 14y + 85 = 0$$

$$\text{Step 1: } x^2 + 12x + y^2 - 14y = -85$$

$$\text{Step 2: } x^2 + 12x + 36 + y^2 - 14y + 49 = -85 + 36 + 49$$

$$\text{Step 3: } (x+6)^2 + (y-7)^2 = 0$$

$$\text{Center } (-6, 7); r = \sqrt{0}$$

$$r = 0.$$

* Since $r=0$, we have only the point $(-6, 7)$ and not a circle.

$$g) x^2 + y^2 - 4x + 2y + 6 = 0$$

$$\underline{\text{Step 1}}: x^2 - 4x + y^2 + 2y = -6$$

$$\underline{\text{Step 2}}: x^2 - 4x + 4 + y^2 + 2y + 1 = -6 + 4 + 1$$

$$\underline{\text{Step 3}}: (x-2)^2 + (y+1)^2 = -1$$

* We can stop here since $r^2 \neq -1$!
This is not the equation of a circle or a point.

$$h) 2x^2 + 2y^2 + 20x - 12y + 18 = 0.$$

EXTRA STEP: Divide each term by 2.

$$x^2 + y^2 + 10x - 6y + 9 = 0$$

$$\text{Step 1: } x^2 + 10x + y^2 - 6y = -9$$

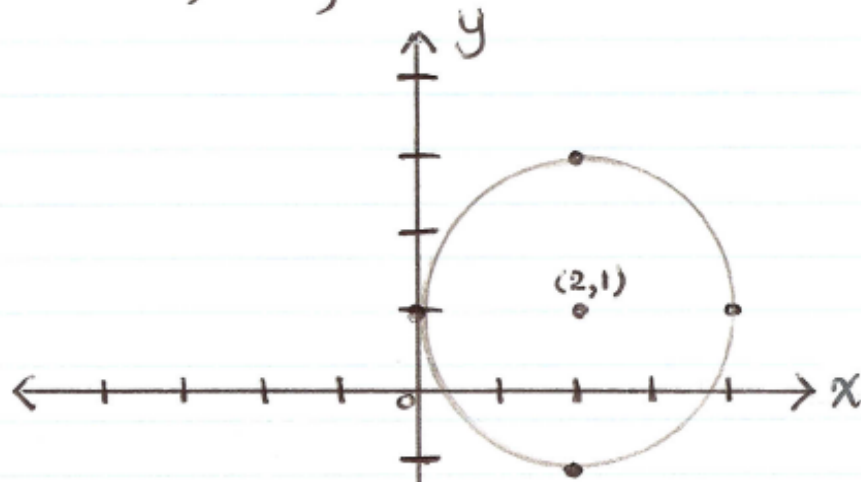
$$\text{Step 2: } x^2 + 10x + 25 + y^2 - 6y + 9 = -9 + 25 + 9$$

$$\text{Step 3: } (x+5)^2 + (y-3)^2 = 25$$

$$\text{Center } (-5, 3); r = \sqrt{25}$$
$$r = 5 \text{ units.}$$

3.

2(c) Center $(2, 1)$; $r = 2$ units.



Domain:

$$\{x \mid 0 \leq x \leq 4, x \in \mathbb{R}\}$$

Range:

$$\{y \mid -1 \leq y \leq 3, y \in \mathbb{R}\}$$

2(h) Center $(-5, 3)$; $r = 5$ units

