

SOLUTIONS \Rightarrow EQUATION OF AN ELLIPSE #2

1a)

$$25(x+1)^2 + 36(y+2)^2 = 900$$

$$\frac{25(x+1)^2}{900} + \frac{36(y+2)^2}{900} = \frac{900}{900}$$

$$\frac{(x+1)^2}{36} + \frac{(y+2)^2}{25} = 1$$

$36 \triangleleft_{\text{MAJOR}}$ $25 \triangleleft_{\text{MINOR}}$

- Center $(-1, -2)$
- Major Axis is Parallel to the x -axis.

$$b) \quad 4(x+2)^2 + 9(y+3)^2 = 36$$

$$\frac{4(x+2)^2}{36} + \frac{9(y+3)^2}{36} = \frac{36}{36}$$
$$\frac{(x+2)^2}{9} + \frac{(y+3)^2}{4} = 1$$

9 ↙ MAJOR ↘ 4 MINOR.

- Center $(-2, -3)$
- Major Axis is Parallel to the x-axis.

$$2. \quad a) \quad 4x^2 + 9y^2 + 8x + 36y - 68 = 0$$

$$\text{Step 1:} \quad 4x^2 + 8x + 9y^2 + 36y = 68$$

$$\text{Extra Step:} \quad 4(x^2 + 2x) + 9(y^2 + 4y) = 68$$

$$\text{Step 2:} \quad 4(x^2 + 2x + 1) + 9(y^2 + 4y + 4) = 68 + 4 + 36$$

$$\text{Step 3:} \quad 4(x+1)^2 + 9(y+2)^2 = 108$$

$$\text{Rearranging:} \quad \frac{4(x+1)^2}{108} + \frac{9(y+2)^2}{108} = \frac{108}{108}$$
$$\frac{(x+1)^2}{27} + \frac{(y+2)^2}{12} = 1 \quad (\text{standard form})$$

$$b) 4x^2 + 9y^2 - 16x - 18y = 11$$

$$\text{Step 1: } 4x^2 - 16x + 9y^2 - 18y = 11$$

$$\text{Extra Step: } 4(x^2 - 4x) + 9(y^2 - 2y) = 11$$

$$\text{Step 2: } 4(x^2 - 4x + 4) + 9(y^2 - 2y + 1) = 11 + 16 + 9$$

$$\text{Step 3: } 4(x-2)^2 + 9(y-1)^2 = 36$$

$$\text{Rearranging: } \frac{4(x-2)^2}{36} + \frac{9(y-1)^2}{36} = \frac{36}{36}$$
$$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{4} = 1 \quad (\text{Standard form})$$

$$3. \frac{(x+3)^2}{36} + \frac{(y+1)^2}{64} = 1$$

a) Center $(-3, -1)$

$$b) \frac{(x+3)^2}{(6)^2} + \frac{(y+1)^2}{(8)^2} = 1$$

Vertices $\Rightarrow (-3, -1-8)$ and $(-3, -1+8)$
 $(-3, -9)$ and $(-3, 7)$

$$4x^2 + 9y^2 - 8x - 18y - 23 = 0$$

$$\text{Step 1: } 4x^2 - 8x + 9y^2 - 18y = 23$$

$$\text{Extra Step: } 4(x^2 - 2x) + 9(y^2 - 2y) = 23$$

$$\text{Step 2: } 4(x^2 - 2x + 1) + 9(y^2 - 2y + 1) = 23 + 4 + 9$$

$$\text{Step 3: } 4(x-1)^2 + 9(y-1)^2 = 36$$

$$\text{Rearranging: } \frac{4(x-1)^2}{36} + \frac{9(y-1)^2}{36} = \frac{36}{36}$$
$$\frac{(x-1)^2}{9} + \frac{(y-1)^2}{4} = 1$$

a) Center $(1, 1)$

b) $\frac{(x-1)^2}{(3)^2} + \frac{(y-1)^2}{(2)^2} = 1$

Vertices. $\Rightarrow (1-3, 1)$ and $(1+3, 1)$
 $(-2, 1)$ and $(4, 1)$

c) Major Axis = $2a$ Minor Axis = $2b$
 = $2(3)$ = $2(2)$
 = 6 units = 4 units.

$$5. 25x^2 + 4y^2 + 100x - 16y + 16 = 0$$

$$\text{Step 1: } 25x^2 + 100x + 4y^2 - 16y = -16$$

$$\text{Extra Step: } 25(x^2 + 4x) + 4(y^2 - 4y) = -16$$

$$\text{Step 2: } 25(x^2 + 4x + 4) + 4(y^2 - 4y + 4) = -16 + 100 + 16$$

$$\text{Step 3: } 25(x+2)^2 + 4(y-2)^2 = 100$$

$$\text{Rearranging: } \frac{25(x+2)^2}{100} + \frac{4(y-2)^2}{100} = \frac{100}{100}$$
$$\frac{(x+2)^2}{4} + \frac{(y-2)^2}{25} = 1$$

$$a) \text{ Center } (-2, 2)$$

$$b) \frac{(x+2)^2}{(2)^2} + \frac{(y-2)^2}{(5)^2} = 1$$

^ MAJOR.

$$\text{Vertices } \Rightarrow (-2, 2-5) \text{ and } (-2, 2+5)$$

$$(-2, -3) \text{ and } (-2, 7)$$

$$c) \text{ Major Axis} = 2a = 2(5) = 10 \text{ units}$$

$$\text{Minor Axis} = 2b = 2(2) = 4 \text{ units}$$