Questions from Homework

④b) $f(x) = \frac{2x-1}{4x}$ $\quad f(x+h) = \frac{2(x+h)-1}{4(x+h)}$

$= \frac{2x+2h-1}{4x+4h}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2x+2h-1}{4x+4h} - \frac{2x-1}{4x}}{h} \cdot \frac{4x(4x+4h)}{4x(4x+4h)}$$

$$= \lim_{h\to 0} \frac{4x(2x+2h-1) - (2x-1)(4x+4h)}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{8x^2+8xh-4x-(8x^2+8xh-4x-4h)}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{4h}{h(4x)(4x+4h)} = \frac{4}{(4x)(4x)} = \frac{4}{16x^2} = \boxed{\frac{1}{4x^2}}$$

⑥b) $y = 2x^2 - 6\sqrt{x}$ at $(4, 20)$ (x_1, y_1)

$y = 2x^2 - 6x^{1/2}$

① Derivative

$y' = 4x - 3x^{-1/2}$

$= 4x - \frac{3}{\sqrt{x}}$

② Sub in x-value:

$y' = 4(4) - \frac{3}{\sqrt{4}}$

$= 16 - \frac{3}{2}$

$= \frac{32-3}{2} = \boxed{\frac{29}{2}}$ ("m")

③ Find equation:

$y - y_1 = m(x - x_1)$

$y - 20 = \frac{29}{2}(x - 4)$

$2 \cdot \quad 2 \cdot \quad 2 \cdot \quad 2 \cdot$

$2y - 40 = 29x - 116$

$\boxed{0 = 29x - 2y - 76}$

Warm Up

Differentiate the following:

$$f(x) = -4x^2 - 5x(x^3+7)^2 + 2\sqrt[5]{x^9} - \frac{5}{x^{10}} + \frac{7x^2}{\sqrt{x}}$$

$= -4x^2 - 5x(x^6+14x^3+49) + 2x^{9/5} - 5x^{-10} + 7x^2(x^{-1/2})$

$= -4x^2 - 5x^7 - 70x^4 - 245x + 2x^{9/5} - 5x^{-10} + 7x^{3/2}$

$F'(x) = -8x - 35x^6 - 280x^3 - 245 + \frac{18}{5}x^{4/5} + 50x^{-11} + \frac{21}{2}x^{1/2}$

$= -8x - 35x^6 - 280x^3 - 245 + \frac{18\sqrt[5]{x^4}}{5} + \frac{50}{x^{11}} + \frac{21\sqrt{x}}{2}$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$f(x)$ = (First) X (Second)

In words, ***the Product Rule*** says that the ***derivative of a product of two functions is: the first function times the derivative of the second function, plus the derivative of the first function times the second function***

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

Examples:

$$y = (2x^3 + 5)(3x^2 - x)$$

$y' = (2x^3+5)(6x-1) + (6x^2)(3x^2-x)$

$= 12x^4 - 2x^3 + 30x - 5 + 18x^4 - 6x^3$

$= 30x^4 - 8x^3 + 30x - 5$

$$f(x) = \sqrt{x}(2 - 3x)$$

$= (x^{1/2})(2-3x)$

$F'(x) = (x^{1/2})(-3) + \left(\frac{1}{2}x^{-1/2}\right)(2-3x)$

$= -3x^{1/2} + x^{-1/2} - \frac{3}{2}x^{1/2}$

$= -\frac{9}{2}x^{1/2} + x^{-1/2}$

$= \frac{1}{\sqrt{x}} - \frac{9\sqrt{x}}{2}$

Examples:

$$f(x)=\left(7x^3-x^2+5\right)\left(x^9+3x-5\right)$$

$$f'(x)=(7x^3-x^2+5)(9x^8+3)+(21x^2-2x)(x^9+3x-5)$$

$$=63x^{11}+21x^3-9x^{10}-3x^2+45x^8+15+21x^{11}+63x^3-105x^2-2x-6x^{10}+10x$$

$$=84x^{11}-11x^{10}+45x^8+84x^3-114x^2+10x+15$$

$$h(t)=\left(t^3-5t\right)\left(6\sqrt{t}-t^{-5}\right)$$

Homework

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