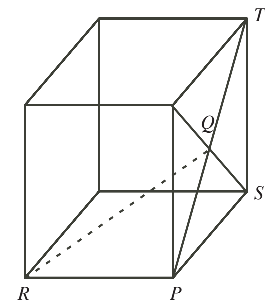
Problem of the Week
Grade 9 and 10

It's a Cube
Solution



Problem

Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 cm. Determine the length of QR.

Solution

Label the corners S and T as shown.

The faces of a cube are squares. The diagonals of a square right bisect each other. It follows that $PQ = QT = \frac{1}{2}PT$. Since the face is a square, $\angle PST = 90°$ and $\triangle PST$ is right angled. Using the Pythagorean Theorem, $PT^2 = PS^2 + ST^2 = 2^2 + 2^2 = 8$ and $PT = \sqrt{8}$. Then $PQ = \frac{1}{2}PT = \frac{\sqrt{8}}{2}$.

Because of the 3-dimensional nature of the problem it may not be obvious to all that $\angle RPQ = 90°$. To help visualize this, note that $\angle RPS = 90°$ because the face of the cube is a square. Rotate PS counterclockwise about point P on the side face of the cube so that the image of PS lies along PQ. The corner angle will not change as a result of the rotation so $\angle RPQ = \angle RPS = 90°$.

We can now use the Pythagorean Theorem in $\triangle RPQ$ to find the length RQ.

$$RQ^2 = RP^2 + PQ^2 = 2^2 + \left(\frac{\sqrt{8}}{2}\right)^2 = 4 + \frac{8}{4} = 4 + 2 = 6 \text{ and } RQ = \sqrt{6} \text{ cm.}$$

$\therefore$ **the length of RQ is $\sqrt{6}$ cm.**

A couple of notes are in order at this point.

First, although the mathematics required to solve this problem was fairly straight forward some students would have found it difficult because of the three dimensional nature of the problem.

Second, we could have simplified $PQ = \frac{1}{2}PT = \frac{\sqrt{8}}{2}$ to $\sqrt{2}$ as follows:

$$\frac{\sqrt{8}}{2} = \frac{\sqrt{4 \times 2}}{2} = \frac{\sqrt{4} \times \sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}.$$

Often simplifying radicals is not a part of the curriculum at the grade 9 or 10 level. The calculation of RQ would have been simpler using $PT = \sqrt{2}$.