Problem of the Week
Grade 9 and 10

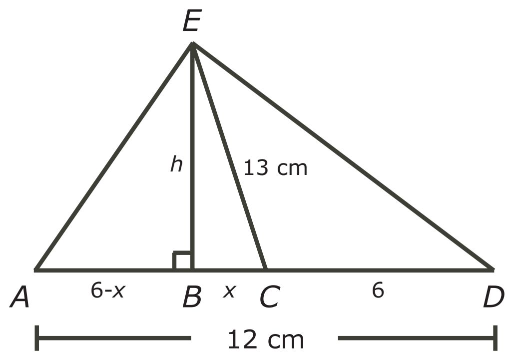
Traversing the Triangle
Solution

Problem

A *median* is a line segment drawn from the vertex of a triangle to the midpoint of the opposite side. A triangle has an area of 72 cm^2. The length of one side is 12 cm and the length of the median to this side is 13 cm. Determine the length of the other two sides of the triangle.

Solution

Start with a diagram to represent the problem.



Let AD be the 12 cm side and EC be the 13 cm median drawn to that side. Draw the altitude from E to side AD meeting it at B. Let EB be h.
Since EC is a median, $AC = CD = \frac{1}{2}(AD) = \frac{1}{2}(12) = 6$ cm.
Let BC, the distance along AD from the altitude to the median, be x.
Then $AB = 6 - x$.

The area of the triangle is 72 cm^2 so $\frac{AD \times EB}{2} = 72$. Then $\frac{12h}{2} = 72$ and $h = 12$ cm follows.

$\triangle EBC$ is right angled so, using Pythagoras' Theorem,
$x^2 = 13^2 - h^2 = 13^2 - 12^2 = 169 - 144 = 25$ and $x = 5$ cm $(x > 0)$.
Then $BD = BC + CD = x + 6 = 11$ cm and $AB = 6 - x = 1$ cm.

$\triangle EAB$ is right angled so, using Pythagoras' Theorem, $EA^2 = EB^2 + AB^2 = 12^2 + 1^2 = 145$ and $EA = \sqrt{145}$ cm. $(EA > 0)$

$\triangle EBD$ is right angled so, using Pythagoras' Theorem,
$ED^2 = EB^2 + BD^2 = 12^2 + 11^2 = 144 + 121 = 265$ and $ED = \sqrt{265}$ cm $(ED > 0)$.

Therefore the lengths of the other two sides are $\sqrt{145}$ cm and $\sqrt{265}$ cm. These lengths are approximately 12.0 cm and 16.3 cm.