

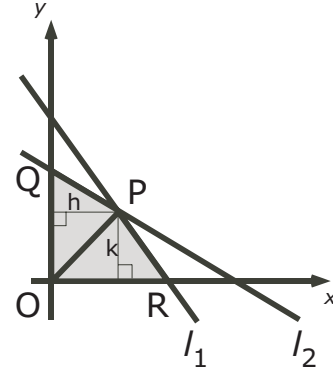


Problem of the Week Grade 9 and 10

A Shady Region Solution

Problem

The shaded region on the diagram is bounded by the lines whose equations are $5x + 2y = 30$, $x + 2y = 22$, $x = 0$, and $y = 0$. Determine the area of the shaded region.



Solution

On the diagram, l_1 represents the line $5x + 2y = 30$ that crosses the x -axis at point R . l_2 represents the line $x + 2y = 22$ which crosses the y -axis at point Q .

Let P represent the point of intersection of l_1 and l_2 . Then h is the horizontal distance from the y -axis to P and k is the vertical distance from the x -axis to P . Let O represent the origin.

To find the x -intercept of l_1 let $y = 0$ in $5x + 2y = 30$. Therefore the x -intercept is 6 and the coordinates of R are $(6, 0)$.

To find the y -intercept of l_2 let $x = 0$ in $x + 2y = 22$. Therefore the y -intercept is 11 and the coordinates of Q are $(0, 11)$.

To find the intersection of l_1 and l_2 , we can use elimination.

$$\begin{array}{rcl} l_1 & 5x + 2y & = 30 \\ l_2 & x + 2y & = 22 \\ \text{Subtracting, we obtain} & 4x & = 8 \\ & \therefore x & = 2 \end{array}$$

Substituting $x = 2$ in l_1 , $10 + 2y = 30$ and $y = 10$. The coordinates of P , the point of intersection, are $(2, 10)$. Therefore, $h = 2$ and $k = 10$. To find the shaded area:

$$\begin{aligned} \text{Area } PQOR &= \text{Area } \triangle PQO + \text{Area } \triangle POR \\ &= \frac{1}{2}h \times OQ + \frac{1}{2}k \times OR \\ &= \frac{1}{2}(2)(11) + \frac{1}{2}(10)(6) \\ &= 11 + 30 \\ &= 41 \end{aligned}$$

Therefore the shaded area is 41 units².

