

SOLUTIONS => EQUATIONS OF CIRCLES/ELLIPSES
REVIEW

1. $\{x^2 + y^2 = r^2\}$

a) $r = 6$
 $\hookrightarrow x^2 + y^2 = (6)^2$
 $x^2 + y^2 = 36$

c) $r = 4\sqrt{7}$
 $\hookrightarrow x^2 + y^2 = (4\sqrt{7})^2$
 $x^2 + y^2 = (16)(7)$
 $x^2 + y^2 = 112$

b) $r = \sqrt{5}$
 $\hookrightarrow x^2 + y^2 = (\sqrt{5})^2$
 $x^2 + y^2 = 5$

d) $r = 3\sqrt{2}$
 $\hookrightarrow x^2 + y^2 = (3\sqrt{2})^2$
 $x^2 + y^2 = (9)(2)$
 $x^2 + y^2 = 18$

2. A: $x^2 + y^2 = 81$

a) radius
 $\hookrightarrow r^2 = 81$
 $r = \sqrt{81}$
 $r = 9$ units

b) x-intercepts
 $\hookrightarrow -9$ and $+9$
c) y-intercepts
 $\hookrightarrow -9$ and 9

d) domain
 $\{x | -9 \leq x \leq 9, x \in \mathbb{R}\}$

e) range
 $\{y | -9 \leq y \leq 9, y \in \mathbb{R}\}$

2. B: $x^2 + y^2 = 48$

a) radius
 $\hookrightarrow r^2 = 48$
 $r = \sqrt{48}$
 $r = \sqrt{16 \times 3}$
 $r = 4\sqrt{3}$ units

c) y-intercepts
 $\hookrightarrow -4\sqrt{3}$ and $4\sqrt{3}$

d) domain
 $\{x | -4\sqrt{3} \leq x \leq 4\sqrt{3}, x \in \mathbb{R}\}$

e) range
 $\{y | -4\sqrt{3} \leq y \leq 4\sqrt{3}, y \in \mathbb{R}\}$

3. $x^2 + y^2 = 25$

a) $(-4, ?)$

If $x = -4$:

$(-4)^2 + y^2 = 25$
 $16 + y^2 = 25$
 $y^2 = 25 - 16$
 $y^2 = 9$
 $y = \pm\sqrt{9}$
 $y = \pm 3$

Coordinate $\Rightarrow (-4, -3)$
or
 $(-4, 3)$

b) $x^2 + y^2 = 25$

$$(?, 3)$$

If $y = 3$

$$x^2 + (3)^2 = 25$$

$$x^2 + 9 = 25$$

$$x^2 = 25 - 9$$

$$x^2 = 16$$

$$x = \pm\sqrt{16}$$

$$x = \pm 4$$

Coordinate: (-4, 3)
or
(4, 3)

c) (5, ?)

If $x = 5$

$$(5)^2 + y^2 = 25$$

$$25 + y^2 = 25$$

$$y^2 = 25 - 25$$

$$y^2 = 0$$

$$y = \pm\sqrt{0}$$

$$y = 0$$

Coordinate: (5, 0)

d) $(2\sqrt{3}, ?)$

If $x = 2\sqrt{3}$

$$(2\sqrt{3})^2 + y^2 = 25$$

$$(4)(3) + y^2 = 25$$

$$12 + y^2 = 25$$

$$y^2 = 25 - 12$$

$$y^2 = 13$$

$$y = \sqrt{13}$$

Coordinate: $(2\sqrt{3}, \sqrt{13})$

4.

EQUATION	CENTER	DOMAIN	RANGE	X-INTERCEPTS	Y-INTERCEPTS
$x^2 + y^2 = 9$	(0, 0)	$\{x -3 \leq x \leq 3, x \in \mathbb{R}\}$	$\{y -3 \leq y \leq 3, y \in \mathbb{R}\}$	-3 and 3	-3 and 3
$x^2 + y^2 = 36$	(0, 0)	$\{x -6 \leq x \leq 6, x \in \mathbb{R}\}$	$\{y -6 \leq y \leq 6, y \in \mathbb{R}\}$	-6 and 6	-6 and 6
$x^2 + y^2 = 121$	(0, 0)	$\{x -11 \leq x \leq 11, x \in \mathbb{R}\}$	$\{y -11 \leq y \leq 11, y \in \mathbb{R}\}$	-11 and 11	-11 and 11

5. a) passing through (2, -4)

↳ If $x = 2$ and $y = -4$

$$x^2 + y^2 = r^2$$

$$(2)^2 + (-4)^2 = r^2$$

$$4 + 16 = r^2$$

$$20 = r^2$$

Therefore, the equation would be:

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 20$$

b) passing through $(4\sqrt{5}, \sqrt{2})$

↳ If $x = 4\sqrt{5}$ and $y = \sqrt{2}$:

$$\begin{aligned}x^2 + y^2 &= r^2 \\(4\sqrt{5})^2 + (\sqrt{2})^2 &= r^2 \\(16)(5) + 2 &= r^2 \\80 + 2 &= r^2 \\82 &= r^2\end{aligned}$$

↳ Therefore, the equation would be :

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 82\end{aligned}$$

c) with an x -intercept of -12°
↳ Point $(-12, 0)$

If $x = -12$ and $y = 0$:

$$\begin{aligned}x^2 + y^2 &= r^2 \\(-12)^2 + (0)^2 &= r^2 \\144 + 0 &= r^2 \\144 &= r^2\end{aligned}$$

↳ Therefore, the equation would be :

$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + y^2 &= 144\end{aligned}$$

6. a) $x^2 + (y-4)^2 = 64$
 $(x-0)^2 + (y-4)^2 = 64$

Center $(0, 4)$

Radius : $r^2 = 64$
 $r = \sqrt{64}$
 $r = 8$ units

b) $(x+2)^2 + y^2 = 49$
 $(x+2)^2 + (y+0)^2 = 49$

Center $(-2, 0)$

Radius : $r^2 = 49$
 $r = \sqrt{49}$
 $r = 7$ units

c) $(x+1)^2 + (y-11)^2 = 100$

Center $(-1, 11)$

Radius : $r^2 = 100$
 $r = \sqrt{100}$
 $r = 10$ units

d) $(x-16)^2 + (y-3)^2 = 144$

Center $(16, 3)$

Radius : $r^2 = 144$
 $r = \sqrt{144}$
 $r = 12$ units

a) $C(-11, 6); r = \sqrt{7}$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x+11)^2 + (y-6)^2 &= (\sqrt{7})^2 \\ (x+11)^2 + (y-6)^2 &= 7\end{aligned}$$

b) $C(0, 3); r = 5$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x-0)^2 + (y-3)^2 &= (5)^2 \\ x^2 + (y-3)^2 &= 25\end{aligned}$$

c) $C(4, -4); r = 13$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x-4)^2 + (y+4)^2 &= (13)^2 \\ (x-4)^2 + (y+4)^2 &= 169\end{aligned}$$

d) $C(-9, 14); r = 2$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x+9)^2 + (y-14)^2 &= (2)^2 \\ (x+9)^2 + (y-14)^2 &= 4\end{aligned}$$

8.a) $C(2, -2)$ and passing through $J(8, 4)$

Method 1:

$$\begin{aligned}D &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(8-2)^2 + (4-2)^2} \\ &= \sqrt{6^2 + 6^2} \\ &= \sqrt{36+36} \\ &= \sqrt{72} \\ &= 6\sqrt{2}\end{aligned}$$

$$r = 6\sqrt{2}; C(2, -2)$$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x-2)^2 + (y+2)^2 &= (6\sqrt{2})^2 \\ (x-2)^2 + (y+2)^2 &= (36)(2) \\ (x-2)^2 + (y+2)^2 &= 72\end{aligned}$$

Method 2:

$$\begin{array}{ccc}C(2, -2) & J(8, 4) \\ h & K & x \\ & & y\end{array}$$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (8-2)^2 + (4-2)^2 &= r^2 \\ 6^2 + 6^2 &= r^2 \\ 36 + 36 &= r^2 \\ 72 &= r^2\end{aligned}$$

b) $C(10, 0)$ and passing through $K(1, -3)$

Method 1:

$$\begin{aligned}D &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(1-10)^2 + (-3-0)^2} \\ &= \sqrt{(-9)^2 + (-3)^2} \\ &= \sqrt{81+9} \\ &= \sqrt{90} \\ &= \sqrt{9 \times 10} \\ &= 3\sqrt{10}\end{aligned}$$

$$r = 3\sqrt{10}; C(10, 0)$$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x-10)^2 + (y-0)^2 &= (3\sqrt{10})^2 \\ (x-10)^2 + y^2 &= (9)(10) \\ (x-10)^2 + y^2 &= 90\end{aligned}$$

Method 2:

$$\begin{array}{ccc}C(10, 0) & K(1, -3) \\ h & K & x \\ & & y\end{array}$$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (1-10)^2 + (-3-0)^2 &= r^2 \\ (-9)^2 + (-3)^2 &= r^2 \\ 81 + 9 &= r^2 \\ 90 &= r^2\end{aligned}$$

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x-10)^2 + (y-0)^2 &= 90 \\ (x-10)^2 + y^2 &= 90\end{aligned}$$

a) $x^2 + y^2 - 12y - 5 = 0$

Step 1: $x^2 + y^2 - 12y = 5$

Step 2: $x^2 + y^2 - 12y + 36 = 5 + 36$

Step 3: $(x-0)^2 + (y-6)^2 = 41$

Center (0, 6); $r = \sqrt{41}$ units

b) $x^2 + y^2 - 4x - 1 = 0$

Step 1: $x^2 - 4x + y^2 = 1$

Step 2: $x^2 - 4x + 4 + y^2 = 1 + 4$

Step 3: $(x-2)^2 + (y-0)^2 = 5$

Center (2, 0); $r = \sqrt{5}$ units

c) $x^2 + y^2 - 6x - 8y - 1 = 0$

Step 1: $x^2 - 6x + y^2 - 8y = 1$

Step 2: $x^2 - 6x + 9 + y^2 - 8y + 16 = 1 + 9 + 16$

Step 3: $(x-3)^2 + (y-4)^2 = 26$

Center (3, 4); $r = \sqrt{26}$ units

d) $2x^2 + 2y^2 + 16x - 8y + 19 = 0$

Extra Step: Divide each term by 2

$$x^2 + y^2 + 8x - 4y + \frac{19}{2} = 0$$

Step 1: $x^2 + 8x + y^2 - 4y = -\frac{19}{2}$

Step 2: $x^2 + 8x + 16 + y^2 - 4y + 4 = -\frac{19}{2} + 16 + 4$

Step 3: $(x+4)^2 + (y-2)^2 = -\frac{19}{2} + \frac{20}{2}$

$$(x+4)^2 + (y-2)^2 = -\frac{19}{2} + \frac{40}{2}$$

$$(x+4)^2 + (y-2)^2 = \frac{21}{2}$$

Center (-4, 2); $r^2 = \frac{21}{2}$

$$r = \sqrt{\frac{21}{2}}$$
 units

e) $3x^2 + 3y^2 - 36x + 48y + 100 = 0$

Extra Step: Divide each term by 3.

$$x^2 + y^2 - 12x + 16y + \frac{100}{3} = 0$$

Step 1: $x^2 - 12x + y^2 + 16y = -\frac{100}{3}$

Step 2: $x^2 - 12x + 36 + y^2 + 16y + 64 = -\frac{100}{3} + 36 + 64$

Step 3: $(x-6)^2 + (y+8)^2 = -\frac{100}{3} + \frac{200}{3}$

$$(x-6)^2 + (y+8)^2 = -\frac{100}{3} + \frac{300}{3}$$

$$(x-6)^2 + (y+8)^2 = \frac{200}{3}$$

Center (6, -8); $r^2 = \frac{200}{3}$

$$r = \sqrt{\frac{200}{3}}$$

$$r = \frac{\sqrt{200} \cdot \sqrt{3}}{3}$$

$$r = \frac{10\sqrt{2}}{\sqrt{3}}$$

10. $x^2 + y^2 - 6x - 8y - 39 = 0$

Step 1: $x^2 - 6x + y^2 - 8y = 39$

Step 2: $x^2 - 6x + 9 + y^2 - 8y + 16 = 39 + 9 + 16$

Step 3: $(x-3)^2 + (y-4)^2 = 64$

a) Center (3, 4)

b) radius: $r^2 = 64$

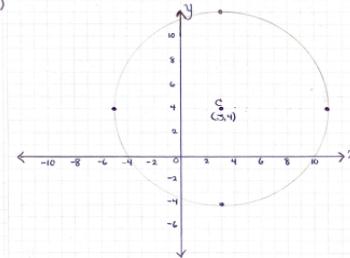
$$r = \sqrt{64}$$

$$r = 8$$
 units

c) Range: $\{y | -4 \leq y \leq 12, y \in \mathbb{R}\}$

d) Domain: $\{x | -5 \leq x \leq 11, x \in \mathbb{R}\}$

e) Range: $\{y | -4 \leq y \leq 12, y \in \mathbb{R}\}$



$$\text{II. a) } \frac{x^2}{36} + \frac{y^2}{1} = 1$$

$$\frac{x^2}{(6)^2} + \frac{y^2}{(1)^2} = 1$$

↳ Horizontal Ellipse

$$a=6$$

$$b=1$$

$$\text{i) Major Axis} = 2a \\ = 2(6) \\ = 12 \text{ units}$$

$$\text{Minor Axis} = 2b \\ = 2(1) \\ = 2 \text{ units}$$

ii) Vertices.
(-6, 0) and (6, 0)

iii) x-ints => -6 and 6
y-ints => -1 and 1

$$\text{b) } \frac{x^2}{16} + \frac{y^2}{49} = 1$$

$$\frac{x^2}{(4)^2} + \frac{y^2}{(7)^2} = 1$$

↳ Vertical Ellipse

$$a=7$$

$$b=4$$

$$\text{i) Major Axis} = 2a \\ = 2(7) \\ = 14 \text{ units}$$

$$\text{Minor Axis} = 2b \\ = 2(4) \\ = 8 \text{ units}$$

ii) Vertices.
(0, -7) and (0, 7)

iii) x-ints => -4 and 4
y-ints => -7 and 7

$$\text{c) } \frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\frac{x^2}{(2)^2} + \frac{y^2}{(3)^2} = 1$$

↳ Vertical Ellipse.

$$a=3$$

$$b=2$$

$$\text{i) Major Axis} = 2a \\ = 2(3) \\ = 6 \text{ units}$$

$$\text{Minor Axis} = 2b \\ = 2(2) \\ = 4 \text{ units}$$

ii) Vertices.
(0, -3) and (0, 3)

iii) x-ints => -2 and 2,
y-ints => -3 and 3

$$\text{12. a) Major Axis is } 12 \Rightarrow 2a=12$$

$$a=6$$

$$\text{Minor Axis is } 5 \Rightarrow 2b=5$$

$$b=\frac{5}{2}$$

*Vertical.

$$\text{Equation: } \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{(\frac{5}{2})^2} + \frac{y^2}{(6)^2} = 1$$

$$\frac{x^2}{\frac{25}{4}} + \frac{y^2}{36} = 1$$

$$\hookrightarrow \frac{4x^2}{25} + \frac{y^2}{36} = 1$$

b) x-ints => ±7

→ y-ints => ±9

$$\text{Equations: } \frac{x^2}{(7)^2} + \frac{y^2}{(9)^2} = 1$$

$$\frac{x^2}{49} + \frac{y^2}{81} = 1$$

c) One vertex => (0, 0)

↳ Remaining vertex must be (-10, 0)

*Therefore the major axis is 20 units
↳ 2a=20
a=10.

$$\rightarrow \text{Minor Axis} = 6$$

$$\hookrightarrow 2b=6$$

$$b=3$$

$$\text{Equation: } \frac{x^2}{(10)^2} + \frac{y^2}{(3)^2} = 1$$

$$\frac{x^2}{100} + \frac{y^2}{9} = 1$$

$$13a) 16(x-1)^2 + 12(y+3)^2 = 48$$

$$\frac{16(x-1)^2}{48} + \frac{12(y+3)^2}{48} = \frac{48}{48}$$

$$\frac{(x-1)^2}{3} + \frac{(y+3)^2}{4} = 1$$

Center $(1, -3)$

Since this ellipse is vertical, it is parallel to the y -axis.

$$b) 3(x+4)^2 + 9(y-2)^2 = 27$$

$$\frac{3(x+4)^2}{27} + \frac{9(y-2)^2}{27} = \frac{27}{27}$$

$$\frac{(x+4)^2}{9} + \frac{(y-2)^2}{3} = 1$$

Center $(-4, 2)$

Since this ellipse is horizontal, it is parallel to the x -axis.

$$14 a) 9x^2 + 25y^2 - 36x - 100y - 89 = 0 \quad x\text{-axis}$$

$$\text{Step 1: } 9x^2 - 36x + 25y^2 - 100y = 89$$

$$\text{Extra Step: } 9(x^2 - 4x) + 25(y^2 - 4y) = 89$$

$$\text{Step 2: } 9(x^2 - 4x + 4) + 25(y^2 - 4y + 4) = 89 + 36 + 100$$

$$\text{Step 3: } 9(x-2)^2 + 25(y-2)^2 = 225$$

$$\frac{9(x-2)^2}{225} + \frac{25(y-2)^2}{225} = \frac{225}{225}$$

$$\frac{(x-2)^2}{25} + \frac{(y-2)^2}{9} = 1$$

$$b) 2x^2 + 5y^2 + 20x - 30y + 75 = 0$$

$$\text{Step 1: } 2x^2 + 20x + 5y^2 - 30y = -75$$

$$\text{Extra Step: } 2(x^2 + 10x) + 5(y^2 - 6y) = -75$$

$$\text{Step 2: } 2(x^2 + 10x + 25) + 5(y^2 - 6y + 9) = -75 + 50 + 45$$

$$\text{Step 3: } 2(x+5)^2 + 5(y-3)^2 = 20$$

$$\frac{2(x+5)^2}{20} + \frac{5(y-3)^2}{20} = \frac{20}{20}$$

$$\frac{(x+5)^2}{10} + \frac{(y-3)^2}{4} = 1$$

$$15. 25x^2 + 16y^2 + 150x + 32y - 159 = 0$$

$$\text{Step 1: } 25x^2 + 150x + 16y^2 + 32y = 159$$

$$\text{Extra Step: } 25(x^2 + 6x) + 16(y^2 + 2y) = 159$$

$$\text{Step 2: } 25(x^2 + 6x + 9) + 16(y^2 + 2y + 1) = 159 + 225 + 16$$

$$\text{Step 3: } 25(x+3)^2 + 16(y+1)^2 = 400$$

$$\frac{25(x+3)^2}{400} + \frac{16(y+1)^2}{400} = \frac{400}{400}$$

$$\frac{(x+3)^2}{16} + \frac{(y+1)^2}{25} = 1$$

a) Center $(-3, -1)$

b) $\frac{(x+3)^2}{(4)^2} + \frac{(y+1)^2}{(5)^2} = 1$ C(-3, -1)

vertices

(-3, -1-5) and (-3, -1+5)

(-3, -6) and (-3, 4)

c) Major Axis = $2a = 2(5) = 10$ units Minor Axis = $2b = 2(4) = 8$ units

16. $4x^2 + 9y^2 - 8x - 54y + 49 = 0$

Step 1: $4x^2 - 8x + 9y^2 - 54y = -49$

Extra Step: $4(x^2 - 2x) + 9(y^2 - 6y) = -49$

Step 2: $4(x^2 - 2x + 1) + 9(y^2 - 6y + 9) = -49 + 4 + 81$

Step 3: $4(x-1)^2 + 9(y-3)^2 = 36$

$$\frac{4(x-1)^2}{36} + \frac{9(y-3)^2}{36} = 1$$

$$\frac{(x-1)^2}{9} + \frac{(y-3)^2}{4} = 1$$

a) Center (1, 3)

b) $\frac{(x-1)^2}{(3)^2} + \frac{(y-3)^2}{(2)^2} = 1$

vertices (1-3, 3) and (1+3, 3)
(-2, 3) and (4, 3)

d) Major Axis = $2a = 2(3) = 6$ units Minor Axis = $2b = 2(2) = 4$ units