SOLUTIONS => EQUATIONS OF CIRCLES / ELLIPSES REVIEW

1 $\boxed{x^2+y^2=r^2}$

a) $r=6$

↳ $x^2+y^2=(6)^2$

$x^2+y^2=36$

b) $r=\sqrt{5}$

↳ $x^2+y^2=(\sqrt{5})^2$

$x^2+y^2=5$

c) $r=4\sqrt{7}$

↳ $x^2+y^2=(4\sqrt{7})^2$

$x^2+y^2=(16)(7)$

$x^2+y^2=112$

d) $r=3\sqrt{2}$

$x^2+y^2=(3\sqrt{2})^2$

$x^2+y^2=(9)(2)$

$x^2+y^2=18$

2. A: $x^2 + y^2 = 81$

a) radius

↳ $r^2 = 81$

$r = \sqrt{81}$

$r = 9$ units

b) x-intercepts

↳ -9 and $+9$

c) y-intercepts

↳ -9 and 9

d) domain

$\{x \mid -9 \leq x \leq 9, x \in R\}$

e) range

$\{y \mid -9 \leq y \leq 9, y \in R\}$

2. B: $x^2+y^2=48$

a) radius

↳ $r^2 = 48$

$r = \sqrt{48}$

$r = \sqrt{16 \times 3}$

$r = 4\sqrt{3}$ units

b) x-intercepts

↳ $-4\sqrt{3}$ and $4\sqrt{3}$

c) y-intercepts

↳ $-4\sqrt{3}$ and $4\sqrt{3}$

d) domain

$\{x \mid -4\sqrt{3} \leq x \leq 4\sqrt{3}, x \in R\}$

e) range

$\{y \mid -4\sqrt{3} \leq y \leq 4\sqrt{3}, y \in R\}$

3. $x^2 + y^2 = 25$

a) $(-4, ?)$

If $x = -4$:

$(-4)^2 + y^2 = 25$

$16 + y^2 = 25$

$y^2 = 25 - 16$

$y^2 = 9$

$y = \pm\sqrt{9}$ $y = \pm 3$

Coordinate => $(-4, -3)$
or
$(-4, 3)$

b) $x^2 + y^2 = 25$

$(?, 3)$

If $y = 3$:

$x^2 + (3)^2 = 25$

$x^2 + 9 = 25$

$x^2 = 25 - 9$

$x^2 = 16$

$x = \pm\sqrt{16}$

$x = \pm 4$

Coordinate: $(-4, 3)$ or $(4, 3)$

c) (5, ?)

If $x = 5$:

$(5)^2 + y^2 = 25$

$25 + y^2 = 25$

$y^2 = 25 - 25$

$y^2 = 0$

$y = \pm\sqrt{0}$

$y = 0$

Coordinate: (5, 0)

d) $(2\sqrt{3}, ?)$

If $x = 2\sqrt{3}$:

$$(2\sqrt{3})^2 + y^2 = 25$$
$$(4)(3) + y^2 = 25$$
$$12 + y^2 = 25$$
$$y^2 = 25 - 12$$
$$y^2 = 13$$
$$y = \pm\sqrt{13}$$

Coordinate: $(2\sqrt{3}, \pm\sqrt{13})$

4.

EQUATION	CENTER	DOMAIN	RANGE	X-INTERCEPTS	Y-INTERCEPTS
$x^2+y^2=9$	$(0,0)$	$\{x \mid -3 \le x \le 3, x \in R\}$	$\{y \mid -3 \le y \le 3, y \in R\}$	-3 and 3	-3 and 3
$x^2+y^2=36$	$(0,0)$	$\{x \mid -6 \le x \le 6, x \in R\}$	$\{y \mid -6 \le y \le 6, y \in R\}$	-6 and 6	-6 and 6
$x^2+y^2=121$	$(0,0)$	$\{x \mid -11 \le x \le 11, x \in R\}$	$\{y \mid -11 \le y \le 11, y \in R\}$	-11 and 11	-11 and 11

5. a) passing through $(2,-4)$

↳ If $x=2$ and $y=-4$:

$$x^2+y^2=r^2$$
$$(2)^2+(-4)^2=r^2$$
$$4+16=r^2$$
$$20=r^2$$

↳ Therefore, the equation would be:

$$x^2+y^2=r^2$$
$$x^2+y^2=20$$

b) passing through $(4\sqrt{5}, \sqrt{2})$

↳ If $x = 4\sqrt{5}$ and $y = \sqrt{2}$:

$$x^2 + y^2 = r^2$$
$$(4\sqrt{5})^2 + (\sqrt{2})^2 = r^2$$
$$(16)(5) + 2 = r^2$$
$$80 + 2 = r^2$$
$$82 = r^2$$

↳ Therefore, the equation would be :

$$x^2 + y^2 = r^2$$
$$x^2 + y^2 = 82$$

c) With an x-intercept of -12:

↳ Point $(-12,0)$

If $x=-12$ and $y=0$:

$$x^2+y^2=r^2$$
$$(-12)^2+(0)^2=r^2$$
$$144+0=r^2$$
$$144=r^2$$

↳ Therefore, the equation would be:

$$x^2+y^2=r^2$$
$$x^2+y^2=144.$$

6. a) $x^2 + (y-4)^2 = 64$

$(x-0)^2 + (y-4)^2 = 64$

Center $(0, 4)$

Radius: $r^2 = 64$

$r = \sqrt{64}$

$r = 8$ units

b) $(x+2)^2 + y^2 = 49$

$(x+2)^2 + (y+0)^2 = 49$

Center $(-2, 0)$

Radius: $r^2 = 49$

$r = \sqrt{49}$

$r = 7$ units

c) $(x+1)^2 + (y-11)^2 = 100$

Center $(-1, 11)$

Radius: $r^2 = 100$

$r = \sqrt{100}$

$r = 10$ units

d) $(x-16)^2 + (y-3)^2 = 144$

Center $(16, 3)$

Radius: $r^2 = 144$

$r = \sqrt{144}$

$r = 12$ units

7.

a) $C(-11,6)$; $r=\sqrt{7}$

$(x-h)^2+(y-k)^2=r^2$
$(x-{-11})^2+(y-6)^2=(\sqrt{7})^2$
$(x+11)^2+(y-6)^2=7$

b) $C(0,3)$; $r=5$

$(x-h)^2+(y-k)^2=r^2$
$(x-0)^2+(y-3)^2=(5)^2$
$x^2+(y-3)^2=25$

c) $C(4,-4)$; $r=13$

$(x-h)^2+(y-k)^2=r^2$
$(x-4)^2+(y-{-4})^2=(13)^2$
$(x-4)^2+(y+4)^2=169$

d) $C(-9, 14)$; $r = 2$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x - -9)^2 + (y - 14)^2 = (2)^2$$
$$(x+9)^2 + (y-14)^2 = 4$$

8.a) $C(2,-2)$ and passing through $J(8,4)$

Method 1:

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(8-2)^2+(4-{-2})^2}$$
$$=\sqrt{(6)^2+(6)^2}$$
$$=\sqrt{36+36}$$
$$=\sqrt{72}$$
$$=\sqrt{36\times 2}$$
$$=6\sqrt{2}$$

$r=6\sqrt{2}$; $C(2,-2)$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-{-2})^2=(6\sqrt{2})^2$$
$$(x-2)^2+(y+2)^2=(36)(2)$$
$$(x-2)^2+(y+2)^2=72$$

Method 2:

$$\underset{h}{C(2},\underset{k}{-2)} \quad J(\underset{x}{8},\underset{y}{4})$$

$$(x-h)^2+(y-k)^2=r^2$$
$$(8-2)^2+(4-{-2})^2=r^2$$
$$(6)^2+(6)^2=r^2$$
$$36+36=r^2$$
$$72=r^2$$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-{-2})^2=72$$
$$(x-2)^2+(y+2)^2=72$$

b) C(10,0) and passing through K(1,-3)

Method 1:

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(1-10)^2+(-3-0)^2}$$
$$=\sqrt{(-9)^2+(-3)^2}$$
$$=\sqrt{81+9}$$
$$=\sqrt{90}$$
$$=\sqrt{9\times10}$$
$$=3\sqrt{10}$$

$r=3\sqrt{10}$; C(10,0)

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-10)^2+(y-0)^2=(3\sqrt{10})^2$$
$$(x-10)^2+y^2=(9)(10)$$
$$(x-10)^2+y^2=90$$

Method 2:

C(10,0) K(1,-3)
h K x y

$$(x-h)^2+(y-k)^2=r^2$$
$$(1-10)^2+(-3-0)^2=r^2$$
$$(-9)^2+(-3)^2=r^2$$
$$81+9=r^2$$
$$90=r^2$$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-10)^2+(y-0)^2=90$$
$$(x-10)^2+y^2=90$$

9a) $x^2+y^2-12y-5=0$

Step 1: $x^2 \quad +y^2-12y=5$

Step 2: $x^2 \quad +y^2-12y+36=5+36$

Step 3: $(x-0)^2+(y-6)^2=41$

Center $(0,6)$; $r=\sqrt{41}$ units

b) $x^2+y^2-4x-1=0$

Step 1: $x^2-4x \quad +y^2=1$

Step 2: $x^2-4x+4 \quad +y^2=1+4$

Step 3: $(x-2)^2+(y-0)^2=5$

Center $(2,0)$; $r=\sqrt{5}$ units

c) $x^2+y^2-6x-8y-1=0$

Step 1: $x^2-6x \quad +y^2-8y=1$

Step 2: $x^2-6x+9+y^2-8y+16=1+9+16$

Step 3: $(x-3)^2+(y-4)^2=26$

Center $(3,4)$; $r=\sqrt{26}$ units

d) $2x^2+2y^2+16x-8y+19=0$

Extra Step: Divide each term by 2

$$x^2+y^2+8x-4y+\frac{19}{2}=0$$

Step 1: $x^2+8x \quad + y^2-4y = -\frac{19}{2}$

Step 2: $x^2+8x+16+y^2-4y+4 = -\frac{19}{2}+16+4$

Step 3: $(x+4)^2+(y-2)^2 = -\frac{19}{2}+\frac{20}{1}$

$$(x+4)^2+(y-2)^2 = -\frac{19}{2}+\frac{40}{2}$$

$$(x+4)^2+(y-2)^2 = \frac{21}{2}$$

Center $(-4,2)$; $r^2=\frac{21}{2}$

$r=\sqrt{\frac{21}{2}}$ units

e) $3x^2+3y^2-36x+48y+100=0$

Extra Step: Divide each term by 3.

$$x^2+y^2-12x+16y+\frac{100}{3}=0$$

Step 1: $x^2-12x \quad +y^2+16y = -\frac{100}{3}$

Step 2: $x^2-12x+36+y^2+16y+64=-\frac{100}{3}+36+64$

Step 3: $(x-6)^2+(y+8)^2=-\frac{100}{3}+\frac{100}{1}$

$$(x-6)^2+(y+8)^2=-\frac{100}{3}+\frac{300}{3}$$

$$(x-6)^2+(y+8)^2=\frac{200}{3}$$

Center $(6,-8)$; $r^2=\frac{200}{3}$

$$r=\sqrt{\frac{200}{3}}$$

$$r=\frac{\sqrt{100\times 2}}{\sqrt{3}}$$

$$r=\frac{10\sqrt{2}}{\sqrt{3}} \text{ units}$$

10. $x^2+y^2-6x-8y-39=0$

Step 1: $x^2-6x \quad +y^2-8y=39$

Step 2: $x^2-6x+9+y^2-8y+16=39+9+16$

Step 3: $(x-3)^2+(y-4)^2=64$

a) Center $(3,4)$

b) radius: $r^2=64$

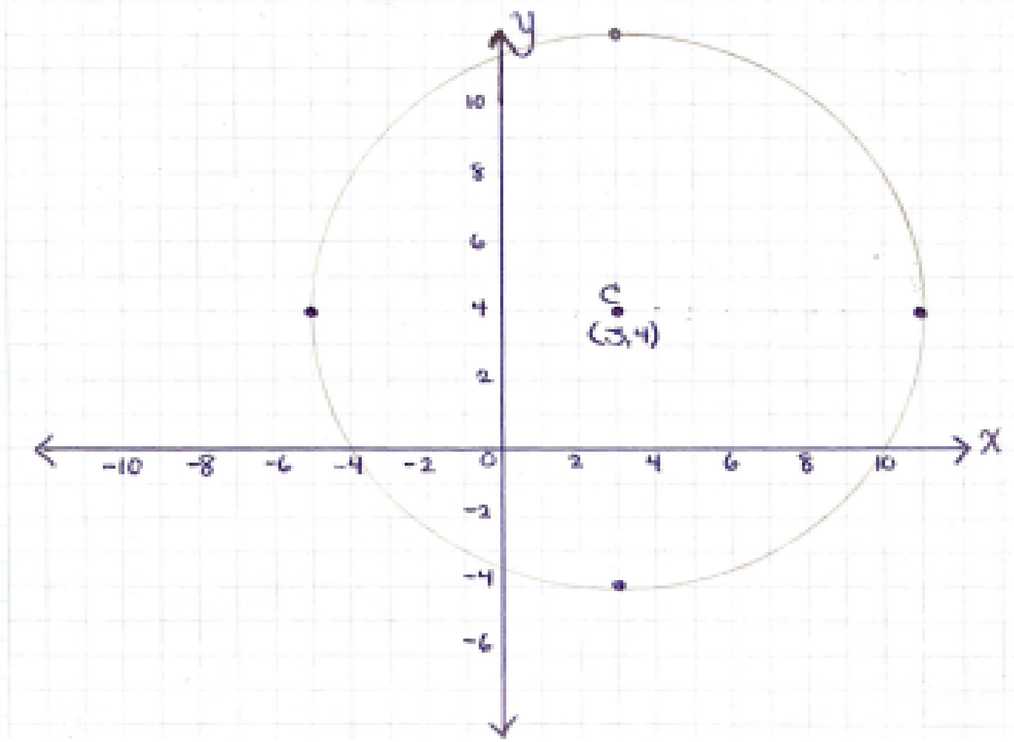
$r=\sqrt{64}$

$r=8$ units

d) Domain: $\{x \mid -5 \le x \le 11, x \in R\}$

e) Range: $\{y \mid -4 \le y \le 12, y \in R\}$

c)



11. a) $\frac{x^2}{36} + \frac{y^2}{1} = 1$

$\frac{x^2}{(6)^2} + \frac{y^2}{(1)^2} = 1$

↳ Horizontal Ellipse

$a = 6$
$b = 1$

i) Major Axis $= 2a$
$= 2(6)$
$= 12$ units

Minor Axis $= 2b$
$= 2(1)$
$= 2$ units

ii) Vertices.
$(-6, 0)$ and $(6, 0)$

iii) x-ints => -6 and 6
y-ints => -1 and 1

b) $\frac{x^2}{16} + \frac{y^2}{49} = 1$

$\frac{x^2}{(4)^2} + \frac{y^2}{(7)^2} = 1$

↳ Vertical Ellipse

$a = 7$
$b = 4$

i) Major Axis $= 2a$
$= 2(7)$
$= 14$ units

Minor Axis $= 2b$
$= 2(4)$
$= 8$ units

ii) vertices.
$(0, -7)$ and $(0, 7)$

iii) x-ints => -4 and 4
y-ints => -7 and 7

c) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$\frac{x^2}{(2)^2} + \frac{y^2}{(3)^2} = 1$

↳ Vertical Ellipse.

$a = 3$
$b = 2$

i) Major Axis $= 2a$
$= 2(3)$
$= 6$ units

Minor Axis $= 2b$
$= 2(2)$
$= 4$ units

ii) vertices
$(0, -3)$ and $(0, 3)$

iii) x-ints $\Rightarrow -2$ and 2
y-ints $\Rightarrow -3$ and 3

12.

a) Major Axis is 12 $\Rightarrow$ $2a=12$

$a=6$

Minor Axis is 5 $\Rightarrow$ $2b=5$

$b=\frac{5}{2}$

* Vertical.

Equation: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$\frac{x^2}{(5/2)^2}+\frac{y^2}{(6)^2}=1$

$\frac{x^2}{25/4}+\frac{y^2}{36}=1$

$\hookrightarrow$ $\frac{4x^2}{25}+\frac{y^2}{36}=1$

b) → x-ints => ± 7

→ y-ints => ± 9.

Equation: $\frac{x^2}{(7)^2} + \frac{y^2}{(9)^2} = 1$

$\frac{x^2}{49} + \frac{y^2}{81} = 1$

c) → One vertex => (10,0)

↳ Remaining vertex must be (-10,0)

* Therefore the major axis is 20 units

↳ $2a = 20$

$a = 10$.

→ Minor Axis = 6

↳ $2b = 6$

$b = 3$

Equation: $\frac{x^2}{(10)^2} + \frac{y^2}{(3)^2} = 1$

$\frac{x^2}{100} + \frac{y^2}{9} = 1$

13a) $16(x-1)^2+12(y+3)^2=48$

$$\frac{16(x-1)^2}{48}+\frac{12(y+3)^2}{48}=\frac{48}{48}$$

$$\frac{(x-1)^2}{3}+\frac{(y+3)^2}{4}=1$$

Center $(1,-3)$

Since this ellipse is vertical, it is parallel to the y-axis.

b) $3(x+4)^2+9(y-2)^2=27$

$$\frac{3(x+4)^2}{27}+\frac{9(y-2)^2}{27}=\frac{27}{27}$$

$$\frac{(x+4)^2}{9}+\frac{(y-2)^2}{3}=1$$

Center $(-4,2)$

Since this ellipse is horizonal, it is parallel to the x-axis.

14. a) $9x^2+25y^2-36x-100y-89=0$ x-axis.

Step 1: $9x^2-36x+25y^2-100y=89$

Extra Step: $9(x^2-4x)+25(y^2-4y)=89$

Step 2: $9(x^2-4x+4)+25(y^2-4y+4)=89+36+100$

Step 3: $9(x-2)^2+25(y-2)^2=225$

$$\frac{9(x-2)^2}{225}+\frac{25(y-2)^2}{225}=\frac{225}{225}$$

$$\frac{(x-2)^2}{25}+\frac{(y-2)^2}{9}=1$$

b) $2x^2+5y^2+20x-30y+75=0$

Step1: $2x^2+20x \quad +5y^2-30y=-75$

Extra Step: $2(x^2+10x)+5(y^2-6y)=-75$

Step2: $2(x^2+10x+25)+5(y^2-6y+9)=-75+50+45$

Step3: $2(x+5)^2+5(y-3)^2=20$

$$\frac{2(x+5)^2}{20}+\frac{5(y-3)^2}{20}=\frac{20}{20}$$

$$\frac{(x+5)^2}{10}+\frac{(y-3)^2}{4}=1$$

15. $25x^2+16y^2+150x+32y-159=0$

Step 1: $25x^2+150x \quad +16y^2+32y=159$

Extra Step: $25(x^2+6x)+16(y^2+2y)=159$

Step 2: $25(x^2+6x+9)+16(y^2+2y+1)=159+225+16$

Step 3: $25(x+3)^2+16(y+1)^2=400$

$$\frac{25(x+3)^2}{400}+\frac{16(y+1)^2}{400}=\frac{400}{400}$$

$$\frac{(x+3)^2}{16}+\frac{(y+1)^2}{25}=1$$

a) Center $(-3,-1)$

b) $\frac{(x+3)^2}{(4)^2} + \frac{(y+1)^2}{(5)^2} = 1$ $\quad C(-3,-1)$

vertices

$(-3, -1-5)$ and $(-3, -1+5)$

$(-3, -6)$ and $(-3, 4)$

c) Major Axis $= 2a$
$= 2(5)$
$= 10$ units

Minor Axis $= 2b$
$= 2(4)$
$= 8$ units

16. $4x^2+9y^2-8x-54y+49=0$

Step 1: $4x^2-8x \quad +9y^2-54y=-49$

Extra Step: $4(x^2-2x)+9(y^2-6y)=-49$

Step 2: $4(x^2-2x+1)+9(y^2-6y+9)=-49+4+81$

Step 3: $4(x-1)^2+9(y-3)^2=36$

$$\frac{4(x-1)^2}{36}+\frac{9(y-3)^2}{36}=\frac{36}{36}$$

$$\frac{(x-1)^2}{9}+\frac{(y-3)^2}{4}=1$$

a) Center $(1,3)$

b) $\dfrac{(x-1)^2}{(3)^2}+\dfrac{(y-3)^2}{(2)^2}=1$

vertices $(1-3,3)$ and $(1+3,3)$
$(-2,3)$ and $(4,3)$

d) Major Axis $= 2a$
$= 2(3)$
$= 6$ units

Minor Axis $= 2b$
$= 2(2)$
$= 4$ units