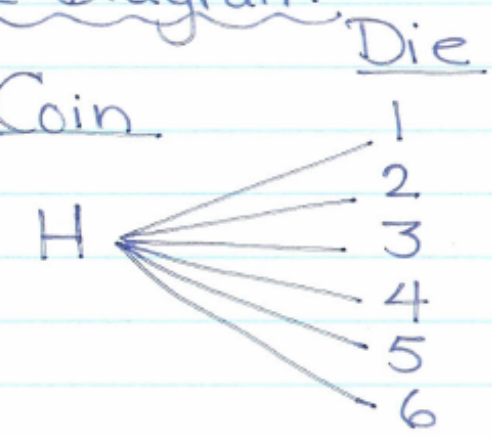
Solutions to Exercise 3.12

1. $P(\text{Event C and Event D}) = P(\text{Event C}) \times P(\text{Event D}) = 0.3 \times 0.15 = 0.045$

2. $P(\text{Event A and Event B}) = P(\text{Event A}) \times P(\text{Event B}) = \frac{2}{7} \times \frac{1}{2} = \frac{2}{14} = \frac{1}{7}$

3. A coin is tossed and a die is rolled.

Tree Diagram



Outcomes

H1
H2
H3
H4
H5
H6

T1
T2
T3
T4
T5
T6

From Tree Diagram ↓

3a) $P(T4) = \frac{1}{12}$

OR

By Calculation only ↓

$P(\text{Tail and } 4) = P(T) \times P(4)$

$= \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{12}$

b) P(Odd and T) $= \frac{3}{12}$ $= \frac{1}{4}$

OR

P(Odd and Tail)
$= P(O) \times P(T)$
$= \frac{3}{6} \times \frac{1}{2}$
$= \frac{3}{12}$
$= \frac{1}{4}$

c) P(Head and Not < 4)
$= \frac{3}{12}$
$= \frac{1}{4}$

OR

P(Head and Not<4)
$= P(H) \times P(Not<4)$
$= \frac{1}{2} \times \frac{3}{6}$
$= \frac{3}{12}$
$= \frac{1}{4}$

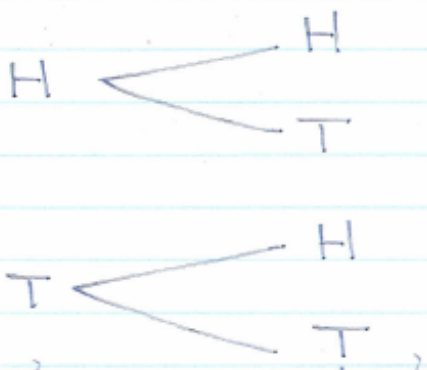
d) P(Neither head nor >3)
= P(Not head and not >3)
$= \frac{3}{12}$
$= \frac{1}{4}$

OR

P(Not head and not >3)
$= P(Not\ head) \times P(not>3)$
$= \frac{1}{2} \times \frac{3}{6}$
$= \frac{3}{12}$
$= \frac{1}{4}$

4. A coin is tossed twice.

Tree Diagram



Outcomes

HH

HT

TH

TT

(This side Tree Diagram) (This side Calculation)

a) $P(HH) = \frac{1}{4}$ OR $P(H \text{ and } H) = P(H) \times P(H)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

b) $P(1T \text{ and then } 1H) = \frac{1}{4}$ OR $P(H \text{ and } T) = P(H) \times P(T)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

c) $P(\text{not TT}) = \frac{3}{4}$ OR $P(\text{not TT}) = 1 - P(\text{TT})$

$= 1 - [P(T) \times P(T)]$

$= 1 - \left[\frac{1}{2} \times \frac{1}{2}\right]$

$= 1 - \frac{1}{4}$

$= \frac{4}{4} - \frac{1}{4}$

$= \frac{3}{4}$

This again demonstrates the power of the tree diagram!

5. A card is chosen and a coin is tossed.

a) P(Ace and Head)
$= P(A) \times P(H)$
$= \frac{4}{52} \times \frac{1}{2}$
$= \frac{4}{104}$
$= \frac{1}{26}$

b) P(Tail and not Club)
$= P(T) \times P(\overline{club})$
$= \frac{1}{2} \times \frac{39}{52}$
$= \frac{39}{104}$
$= \frac{3}{8}$

c) P(neither choosing a tail nor choosing red card)
$= P(\text{not tail and not red})$
$= P(\overline{\text{tail}}) \times P(\overline{\text{red}})$
$= \frac{1}{2} \times \frac{26}{52}$
$= \frac{26}{104}$
$= \frac{1}{4}$

6. A card is chosen from 52 cards.

a) It is important to know that a card is replaced in the deck before the next card is chosen to show that the choices are independent of each other.

b) (i) $P(\text{red and ace}) = P(\text{red}) \times P(\text{ace})$

$= \frac{26}{52} \times \frac{4}{52}$

$= \frac{104}{2704}$

$= \frac{1}{26}$

(ii) $P(\text{queen and even}) = P(\text{queen}) \times P(\text{even})$

$= \frac{4}{52} \times \frac{20}{52}$

$= \frac{80}{2704}$

$= \frac{5}{169}$

(iii) P(face card and King of spades)

$= P(\text{face card}) \times P(\text{King of spades})$

$= \frac{12}{52} \times \frac{1}{52}$

$= \frac{12}{2704}$

$= \frac{3}{676}$

c) P(not ace and not ace)

$= 1 - P(\text{ace and ace})$

$= 1 - [P(\text{ace}) \times P(\text{ace})]$

$= 1 - \left[\frac{4}{52} \times \frac{4}{52}\right]$

$= 1 - \frac{16}{2704}$

$= \frac{2704}{2704} - \frac{16}{2704}$

$= \frac{2688}{2704}$

$= \frac{168}{169}$

8. Apartment Building ⇒ 10 floors, 8 apts/floor

a) P(5th floor and even)
= P(5th floor) x P(even)
= $\frac{1}{10} \times \frac{40}{80}$
= $\frac{40}{800}$
= $\frac{1}{20}$

b) P(above 5th and >6)
= P(above 5th) x P(>6)
= $\frac{5}{10} \times \frac{20}{80}$
= $\frac{1}{2} \times \frac{1}{4}$
= $\frac{1}{8}$

9. 2 : 3

Adults 25 yrs+ ↙ ↘ Adults under 25 yrs

This can be written as:

$\frac{2}{5}$ (Adults 25 yrs+)

$\frac{3}{5}$ (Adults under 25)

a) P(Under 25 and Under 25)
= P(Under 25) x P(Under 25)
= $\frac{3}{5} \times \frac{3}{5}$
= $\frac{9}{25}$

b) P(25 yrs+ and 25 yrs+ and Under 25)
= P(25 yrs+) x P(25 yrs+) x P(Under 25)
= $\frac{2}{5} \times \frac{2}{5} \times \frac{3}{5}$
= $\frac{12}{125}$