Making Connections

$100(0.87)^{1/2}$

Coffee, Tea, and Hot Chocolate contain caffeine.

The expression $100(0.87)^{1/2}$

represents the percent of caffeine
left in your body 1/2 hour after you
drink a caffeine beverage

How can you estimate the value of $0.87^{1/2}$

Let's Take a Closer Look!!

Fill in the chart. (You can use your calculator!!)

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$

x	$x^{\frac{1}{3}}$
1	$1^{\frac{1}{3}} = 1$
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	$64^{\frac{1}{3}} = 4$
125	$125^{\frac{1}{3}} = 5$

What do you notice?

In Grade 9

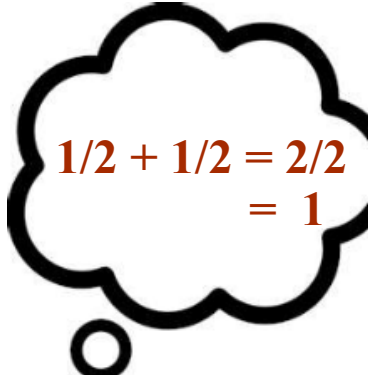To **multiply powers** with the same base you **add.**

$$a^m \text{ x } a^n = a^{m+n}$$

Examples:

1. $5^3 \text{ x } 5^2 = 5^5$

2. $8^5 \text{ x } 8^2 = 8^7$

3. $4^4 \text{ x } 4^2 = 4^6$

$$5^{1/2} \text{ x } 5^{1/2} = 5$$

1/2 + 1/2 = 2/2 = 1

This can also be written like:

$$\sqrt{5} \text{ x } \sqrt{5} = \sqrt{25}$$
$$= 5$$

$$2^{1/3} \text{ x } 2^{1/3} \text{x } 2^{1/3} = 2$$

$$1/3 + 1/3 + 1/3 = 3/3 = 1$$

This can also be written like:

$$\sqrt[3]{2} \text{ x } \sqrt[3]{2} \text{ x } \sqrt[3]{2} = \sqrt[3]{8}$$
$$= 2$$

Our Conclusion

- **Raising a number to an exponent of 1/2 is equivalent to taking the square root!**

- **Raising a number to an exponent of 1/3 is equivalent to taking the cube root!**

$$X^{1/n} = \sqrt[n]{X}$$

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Practice Questions

Calculate each of the following without using a calculator:

$$27^{1/3} \qquad 100^{1/2} \qquad 16^{1/4}$$

$\sqrt[3]{27}$

$= 3$

$\sqrt{100}$

$= 10$

$\sqrt[4]{16}$

$= 2$

Calculate each of the following without using a calculator:

$36^{0.5}$

$36^{\frac{1}{2}}$

$\sqrt{36}$

$= 6$

$32^{0.2}$ $\frac{2}{10}$

$32^{\frac{1}{5}}$

$\sqrt[5]{32}$

$= 2$

$625^{0.25}$ $\frac{25}{100}$

$625^{\frac{1}{4}}$

$\sqrt[4]{625}$

$= 5$

Calculate each of the following without using a calculator:

$4^{3/2}$ ← index, ← cube

$27^{2/3}$ ← square, ← cube root

$\left(\sqrt{4}\right)^3$

$(2)^3$

$= 8$

$\left(\sqrt[3]{27}\right)^2$

$(3)^2$

$= 9$

Therefore:

$$x^{m/n} = \left(\sqrt[n]{x}\right)^m$$

← power, ← index