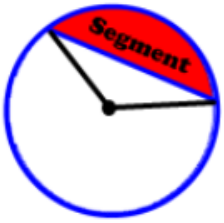
## Area of a Segment




***Reminder:*** A segment of a circle is the region between a chord and the arc subtended by the chord. Any chord bounds two segments, which are different in area except when the chord is a diameter.

### To find the area of a segment:

**Step 1** – Calculate the area of the ***sector*** using:

1. $A = \Pi r^2$

2. $\frac{\text{Sector Area}}{\text{Area of Circle}} = \frac{\text{Angle}}{360^\circ}$

This will give you $A_{sector}$

**Step 2** – Calculate the area of the ***triangle*** using:

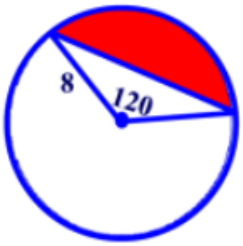
$$A_{triangle} = \tfrac{1}{2}\, r^2 \sin \theta$$

$\theta$ = angle
$r$ = radius

**Step 3** – Calculate the area of the ***segment*** using:

$$A_{segment} = A_{sector} - A_{triangle}$$

## Example Problem:




Find the area of a segment of a circle with a central angle of 120 degrees and a radius of 8 cm.
*Express the answer to two places after the decimal.*

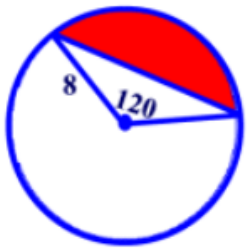
## Solution:

**Step 1** – Calculate the area of the ***sector*** using:

1. $\mathbf{A = \Pi r^2}$
   $= \Pi(8\text{ cm})^2$
   $= \Pi(64\text{ cm}^2)$
   $= 201.06\text{ cm}^2$

2. $$\frac{\textbf{Sector Area}}{\textbf{Area of Circle}} = \frac{\textbf{Angle}}{\textbf{360}^\circ}$$
   $$\frac{\text{Sector Area}}{201.06\text{ cm}^2} = \frac{120^\circ}{360^\circ}$$
   $$\frac{(\text{Sector Area})(360^\circ)}{360^\circ} = \frac{(201.06\text{ cm}^2)(120^\circ)}{360^\circ}$$
   $$A_{sector} = 67.02\text{ cm}^2$$

**Step 2** – Calculate the area of the ***triangle*** using:

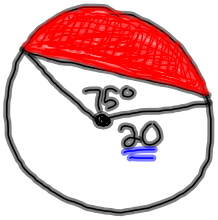



$$A_{triangle} = \tfrac{1}{2} r^2 \sin \theta$$
$$= \tfrac{1}{2} (8 \text{ cm})^2 \sin (120^\circ)$$
$$= \tfrac{1}{2} (64 \text{ cm}^2)(0.8660)$$
$$= \tfrac{1}{2} (55.4240 \text{ cm}^2)$$
$$= 27.7120 \text{ cm}^2 \text{ or } 27.71 \text{ cm}^2$$

**Step 3** – Calculate the area of the ***segment*** using:

$$A_{segment} = A_{sector} - A_{triangle}$$
$$= 67.02 \text{ cm}^2 - 27.71 \text{ cm}^2$$
$$= 39.31 \text{ cm}^2$$

Homework:

① 


Given:
radius = 20cm
angle = $\theta = 75°$

Step 1: a) $A = \pi r^2$

$A = \pi(20)^2$

$A = 1256.64 cm^2$

(ii) $\frac{\text{Sector Area}}{\text{Area of Circle}} = \frac{\theta}{360}$

$\frac{\text{Sector Area}}{1256.64} = \frac{75}{360}$

$\frac{360(SA)}{360} = \frac{94248}{360}$

Sector Area $= 261.8 cm^2$

Step 2: $A_\Delta = \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2}(20)^2 \sin(75°)$

$= \frac{1}{2}(400)\sin(75)$

$= \frac{1}{2}(400)(0.965)$

$= 193.18 cm^2$

Step 3: $A_{segment} = A_{sector} - A_{triangle}$

$= 261.8 cm^2 - 193.18 cm^2$

$= 68.62 cm^2$