Problem of the Week
Grade 11 and 12

Partly Salted
Solution

Problem

When one kilogram of salt is added to a solution of salt and water, the solution becomes $\frac{1}{3}$ salt by mass. One kilogram of water is then added to the new solution resulting in a solution that is $\frac{3}{10}$ salt by mass. What fraction of the original solution was salt?

Solution

Let S represent the mass of salt in the original solution.
Let W represent the mass of water in the original solution.
Then $S + W$ represents the total mass of the original solution.

After adding 1 kg of salt to the original solution, there is $(S + 1)$ kg of salt and $(S + W + 1)$ kg of solution. Since this new solution is one third salt by mass,

$$\begin{aligned} \frac{S+1}{S+W+1} &= \frac{1}{3} \\ 3(S+1) &= 1(S+W+1) \\ 3S+3 &= S+W+1 \\ 2S+2 &= W \qquad (1) \end{aligned}$$

After adding 1 kg of water to the new solution, there is still $(S + 1)$ kg of salt in $(S + W + 2)$ kg of solution. Since this new solution is three tenths salt by mass,

$$\begin{aligned} \frac{S+1}{S+W+2} &= \frac{3}{10} \\ 10(S+1) &= 3(S+W+2) \\ 10S+10 &= 3S+3W+6 \\ 7S-3W &= -4 \qquad (2) \end{aligned}$$

Substituting (1) into (2),

$$\begin{aligned} 7S-3(2S+2) &= -4 \\ 7S-6S-6 &= -4 \\ S &= 2 \text{ kg} \end{aligned}$$

Substituting for S in (1), $W = 6$ kg. Then the original solution was $S + W = 8$ kg. The fraction of the original solution that was salt was $\frac{2}{8} = \frac{1}{4}$.

$\therefore$ one quarter of the original solution was salt by mass.