

Solve the following system of equations:

Inconsistent System (planes do not intersect)

$$\begin{array}{l} 2x + y - 3z = -3 \\ x + 3y - 4z = 11 \\ x + 8y - 9z = 5 \end{array} \quad \begin{array}{l} 2x + y - 3z = -3 \\ \Leftrightarrow 2x + 6y - 8z = 22 \\ \hline -5y + 5z = -25 \end{array} \quad \begin{array}{l} x + 3y - 4z = 11 \\ \Leftrightarrow x + 8y - 9z = 5 \\ \hline -5y + 5z = 6 \end{array}$$

$$\begin{array}{l} -5y + 5z = -25 \\ \Leftrightarrow -5y + 5z = 6 \\ \hline 0 = -31 \end{array}$$

No Solution

Consistent System Dependent Solution
(planes intersect along a line)

$$\begin{array}{l} x + y + z = 3 \\ 2x + y + 4z = 8 \\ x + 2y - z = 1 \end{array} \quad \begin{array}{l} x + y + z = 3 \\ \underline{2x + y + 4z = 8} \quad (-) \quad x + 2y - z = 1 \\ -x - 3z = -5 \end{array} \quad \begin{array}{l} 4x + 2y + 8z = 16 \\ \underline{x + 2y - z = 1} \\ 3x + 9z = 15 \end{array}$$

$$\begin{array}{l} -3x - 9z = -15 \\ (+) \quad \underline{3x + 9z = 15} \\ \hline 0 = 0 \end{array}$$

Infinite Solutions

$$\boxed{\text{let } z = t}$$

$$\begin{array}{l} -x - 3z = -5 \\ -x - 3(t) = -5 \\ -x = 3t - 5 \\ x = -3t + 5 \\ \boxed{x = 5 - 3t} \end{array}$$

$$\begin{array}{l} x + y + z = 3 \\ (5 - 3t) + y + t = 3 \\ 5 - 2t + y = 3 \\ \boxed{y = 2t - 2} \end{array}$$

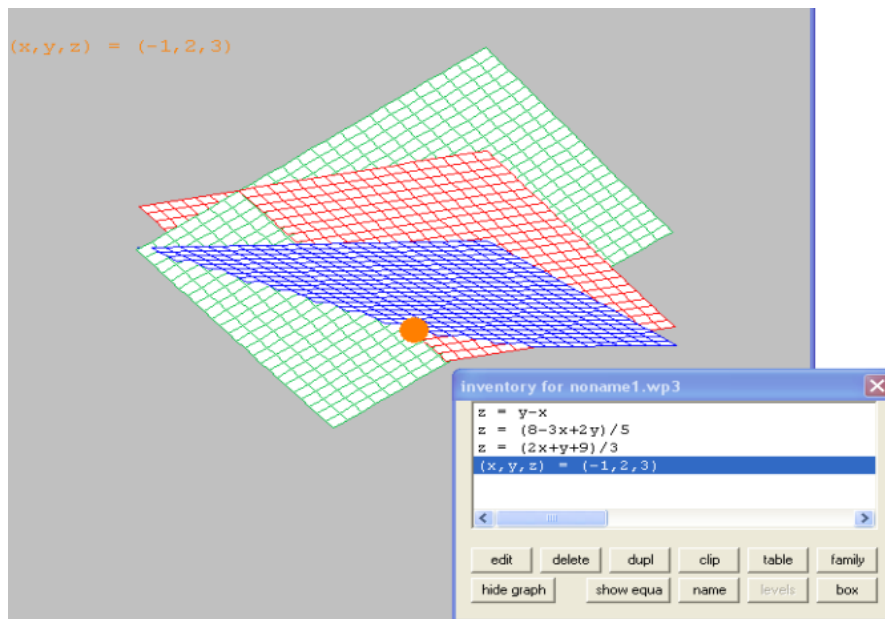
$$(5 - 3t, 2t - 2, t)$$

Types of Systems

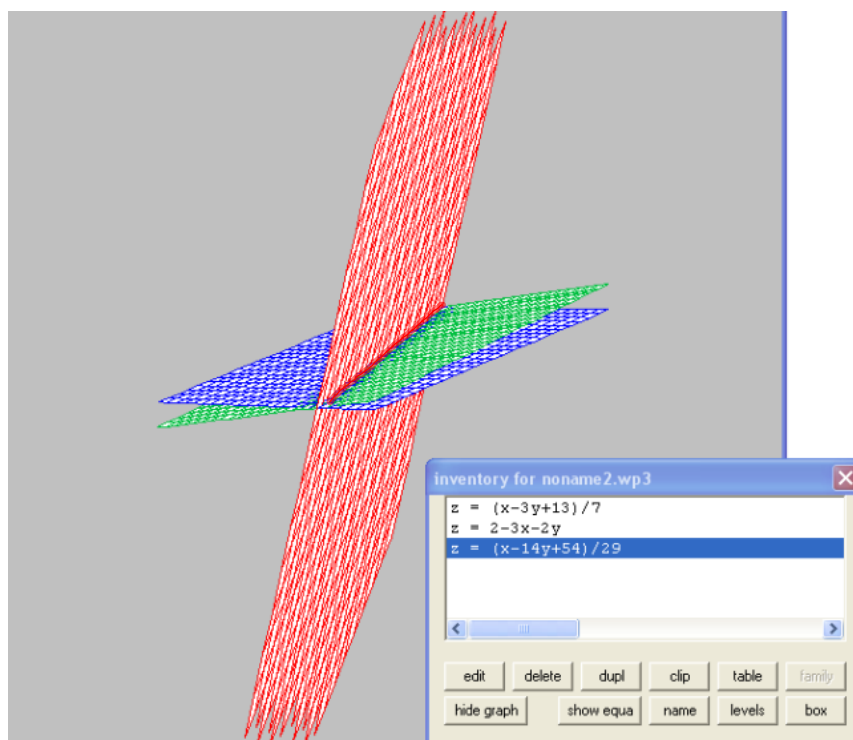
Remember: Looking at **intersecting planes**!

Consistent: At least one solution

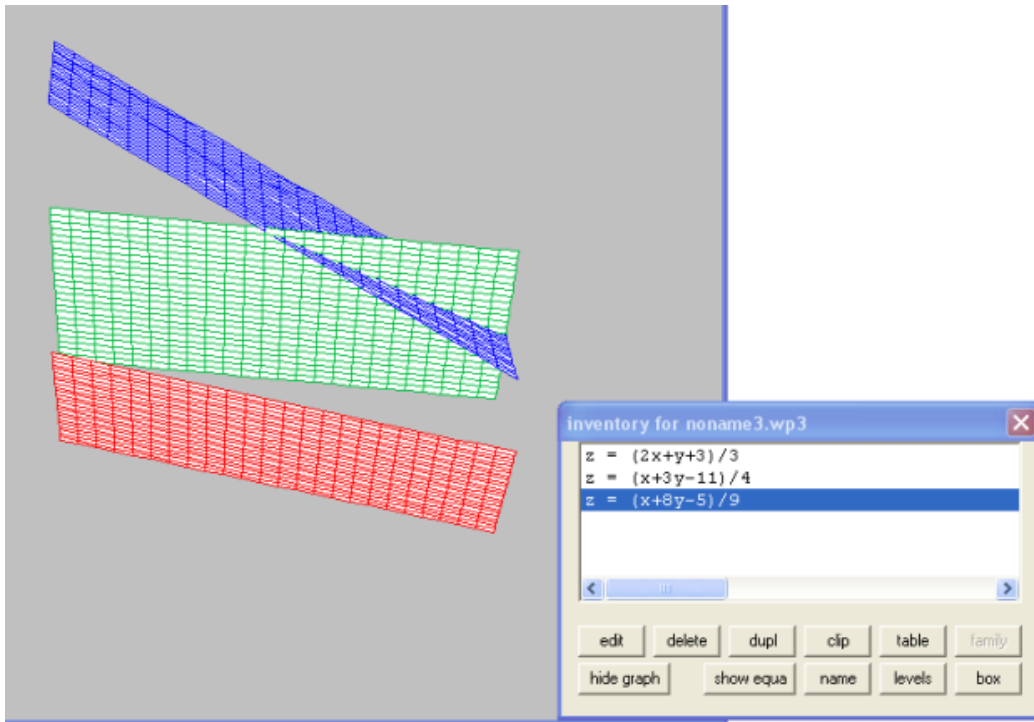
Independent: One Unique solution



Dependent: Infinite number of solutions

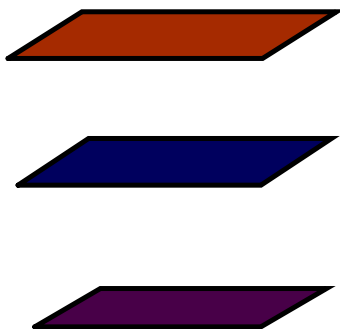


Inconsistent: No Solutions

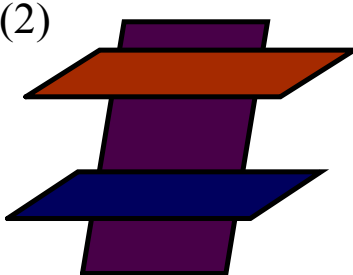


3 Possible Orientations That Give No Solution...

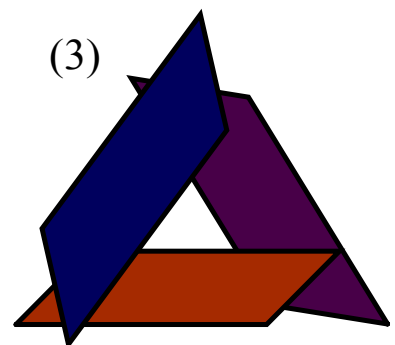
(1)



(2)



(3)



More Word Problems

A hockey stick, gloves, and a helmet cost \$425. The stick costs twice as much as the gloves. The helmet costs \$25 more than the gloves. Find the cost of each

Let x = cost of the stick

Let y = cost of the gloves

Let z = cost of the helmet

$$x = 2y$$

$$z = y + 25$$

$$\textcircled{1} \quad x + y + z = 425 \quad x + y + z = 425$$

$$\textcircled{2} \quad x - 2y = 0 \quad (\rightarrow) \quad x - 2y = 0$$

$$\textcircled{3} \quad -y + z = 25 \quad \textcircled{4} \quad 3y + z = 425$$

$$-y + z = 25$$

$$\rightarrow \frac{3y + z = 425}{-y + z = 25}$$

$$-4y = -400$$

$$y = \$100$$

$$3y + z = 425$$

$$3(100) + z = 425$$

$$300 + z = 425$$

$$z = \$125$$

$$x - 2y = 0$$

$$x - 2(100) = 0$$

$$x - 200 = 0$$

$$x = \$200$$

An ipod, a docking station, and an itunes subscription costs \$825. The ipod costs \$150 more than the docking station. The station costs 4 times the amount of the subscription. Find the cost of each.

Let $x = \text{ipod} (\$)$
Let $y = \text{d.s.} (\$)$
Let $z = \text{itunes} (\$)$

① $x + y + z = 825$ $x + y + z = 825$

② $x - y = 150$ $\Leftrightarrow x - y = 150$

③ $y - 4z = 0$ ④ $2y + z = 675$

$2y - 8z = 0$
 $\Leftrightarrow \frac{2y + z = 675}{-9z = -675}$
 $z = 75$

$y - 4(75) = 0$
 $y - 300 = 0$
 $y = 300$

$x - (300) = 150$
 $x = 450$

Homework