Matrices

Cleveland invested a total of $11,000 with three banks at interest rates of 8%, 10%, and 16% per year. His total annual interest earned was $1,420. If his investment at the 16% bank was twice as much as his investment at the 10% bank, how much did he invest at all three banks?

Let x = investment @ 8%

Let y = investment @ 10% $z = 2y$

Let z = investment @ 16%

$$x + y + z = 11000$$

$$0.08x + 0.10y + 0.16z = 1420$$

$$-2y + z = 0$$

$$x + y + z = 11000$$

$$8x + 10y + 16z = 142000$$

$$\boxed{-2y + z = 0}$$

$$8x + 8y + 8z = 88000$$

$$(-)\ 8x + 10y + 16z = 142000$$

$$\boxed{-2y - 8z = -54000}$$

$$-2y + z = 0$$

$$(-)\ -2y - 8z = -54000$$

$$9z = 54000$$

$$\boxed{z = \$6000}$$

$$-2y + z = 0$$

$$-2y + 6000 = 0$$

$$6000 = 2y$$

$$\boxed{\$3000 = y}$$

$$x + y + z = 11000$$

$$x + 3000 + 6000 = 11000$$

$$\boxed{x = \$2000}$$

Matrix - a rectangular array of numbers enclosed in parentheses

Example:

$$\begin{pmatrix} 2 & 1 & 13 \\ 6 & -2 & 8 \end{pmatrix}$$

Each number in a matrix is called an "entry".

A matrix is made up of "rows" and "columns".

The dimensions of a matrix state the size of it.

ROWS X COLUMNS   2 X 3

$$\begin{pmatrix} 2 & 3 & -4 \\ 1 & -2 & \text{⑤} \\ 0 & 4 & 1 \end{pmatrix}$$

The dimensions of the above matrix are: 3×3

The entries in row 2 are: 1, -2, 5

The entries in column 3 are: -4, 5, 1

The entry in row 2 column 3 is: 5

MATRIX OPERATIONS

Adding & Subtracting Matrices ✳

To add and subtract matrices the **_dimensions_** of each matrix **_must_** be the **_same_**.

$$\begin{pmatrix} -3 & 5 \\ 2 & -4 \end{pmatrix} - \begin{pmatrix} 7 & 0 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -10 & 5 \\ -1 & -8 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 4 \\ 1 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 6 & 7 \\ -1 & 11 \end{pmatrix}$$

$$\text{X} + \begin{pmatrix} -3 & 4 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 2 & -8 \\ 3 & -2 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & -8 \\ 3 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 4 \\ 5 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -12 \\ -2 & -9 \end{pmatrix}$$

$$3 \begin{pmatrix} 5 & -2 \\ 4 & 1 \\ -7 & 0 \end{pmatrix} = \begin{pmatrix} 15 & -6 \\ 12 & 3 \\ -21 & 0 \end{pmatrix}$$

Multiply through the matrix!

$$A = \begin{pmatrix} 3 & -1 \\ 0 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} -3 & 0 \\ -1 & -2 \end{pmatrix}$$

Find 3A + 2 B

$$= 3 \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 \\ 0 & 12 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -1 \\ 6 & 12 \end{bmatrix}$$

<u>Matrix Multiplication</u>

Steps:

1. **State the dimensions of each matrix.**

2. **Determine if it is possible to multiply them.**

3. **Set up the "Template".**

4. **Multiply "Row x Column"**

5. **Simplify**

Matrix Multiplication

In order to multiply matrices, the number of columns in the 1st matrix must equal the number of rows in the 2nd matrix.

Product Dimensions:

$$(\# \text{ rows } 1st) \quad x \quad (\# \text{ columns } 2nd)$$

Ex.
$$\begin{pmatrix} 2 & 7 \\ 3 & 5 \end{pmatrix} \quad x \quad \begin{pmatrix} 6 & -2 & 0 & -1 \\ 7 & 1 & 5 & 4 \end{pmatrix} = \begin{pmatrix} - & - & - & - \\ - & - & - & - \end{pmatrix}$$

$$(2 \times 2) \qquad (2 \times 4)$$

Always multiply a row through a column, adding the products as you go.

Ex.

$$2 \times 3 \qquad\qquad 3 \times 2$$

$$\begin{pmatrix} 5 & 1 & -1 \\ 6 & 2 & 4 \end{pmatrix} \quad x \quad \begin{pmatrix} 4 & -1 \\ 2 & -5 \\ -3 & 0 \end{pmatrix}$$

$$= \begin{bmatrix} 20+2+3 & -5+(-5)+0 \\ 24+4+(-12) & -6+(-10)+0 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & -10 \\ 16 & -16 \end{bmatrix}$$

Matrix Multiplication

2.

$\underline{3} \times \underline{2}$ $\underline{2} \times \underline{2}$

$$\begin{pmatrix} 2 & 1 \\ 3 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 1 & 4 \end{pmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 6 \\ 1 & 8 \end{bmatrix}$$

3.

$\underline{1} \times \underline{4}$ $\underline{4} \times \underline{1}$

$$(2 \quad 3 \quad 5 \quad 7) \begin{pmatrix} 5 \\ 0 \\ 2 \\ 0 \end{pmatrix} = \begin{bmatrix} 20 \end{bmatrix}$$

What would the entry for r3c4 be in the product?

4.

5×4

$$\begin{pmatrix} 2 & 3 & 0 & 6 \\ 0 & 2 & 8 & 2 \\ 1 & 0 & 3 & 1 \\ 8 & 4 & 4 & 8 \\ 4 & 7 & 1 & 9 \end{pmatrix} \begin{pmatrix} 6 & 2 & 0 & 2 & 0 \\ 2 & 8 & 8 & 1 & 8 \\ 1 & 3 & 9 & 5 & 2 \\ 5 & 0 & 4 & 3 & 6 \end{pmatrix}$$

4×5

$$= \begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & 20 & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

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Homework