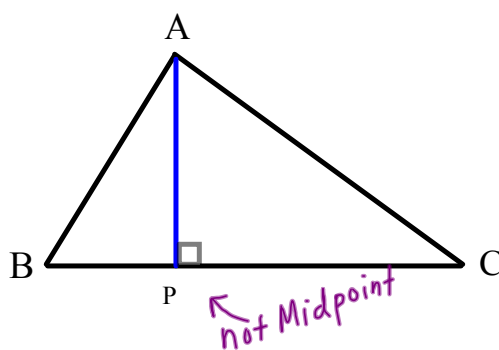


Properties of Triangles

Altitude:

a perpendicular line drawn from a vertex to the opposite side in a triangle.

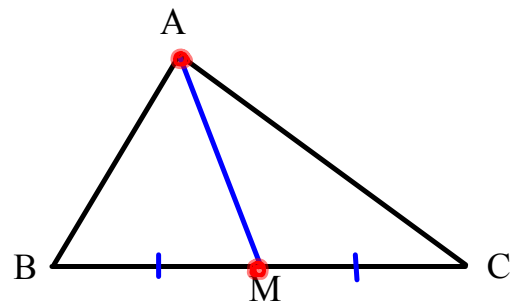


To get equation, find...

- ① - m_{BC}
- ② - m_{\perp} ← slope
- ③ - point A ← point
- ④ **USE: $y - y_1 = m(x - x_1)$**

Median

a line drawn from a vertex to the midpoint of the opposite side in a triangle.



To get equation, find...

- ① - midpoint of BC
- ③ - m_{AM} ← slope
- ③ - point A or M ← point

④ **USE: $y - y_1 = m(x - x_1)$**

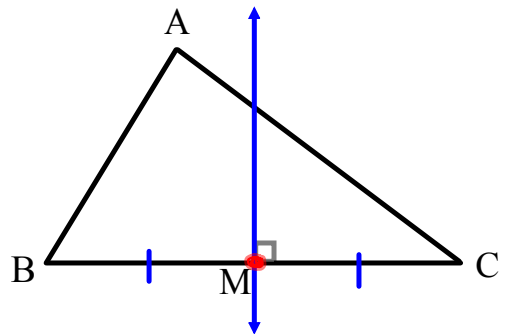
To get length, find...

- ① - midpoint of BC
- ⑥ - D_{AM} ← 2 points

$$D_{AM} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Right Bisector Perpendicular Bisector

a perpendicular line drawn through the midpoint of a line segment.

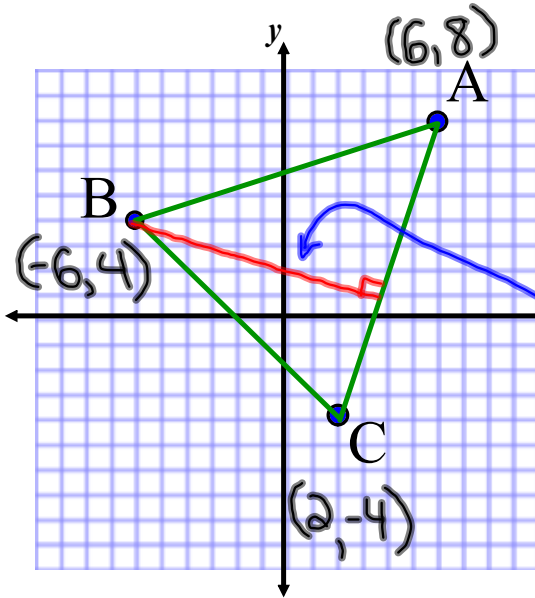


To get equation, find...

- ① • m_{BC}
- ② • $m_{\perp BC}$ ← slope
- ③ • midpoint of BC ← point
- ④ **USE: $y - y_1 = m(x - x_1)$**

Example:

ΔABC has vertices A (6, 8); B(-6, 4) & C(2, -4).



Determine the equation of the altitude from B to AC.

$$\textcircled{1} m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 8}{2 - 6} = \frac{-12}{-4} = 3$$

$$\textcircled{2} m_{\perp} = -\frac{1}{3}$$

$$\textcircled{3} B(-6, 4)$$

$$\textcircled{4} y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{3}(x - 6)$$

$$y - 4 = -\frac{1}{3}(x + 6)$$

$$3 \cdot y - 4 \cdot 3 = -\frac{1}{3} \cdot 3 \cdot x - 2 \cdot 3$$

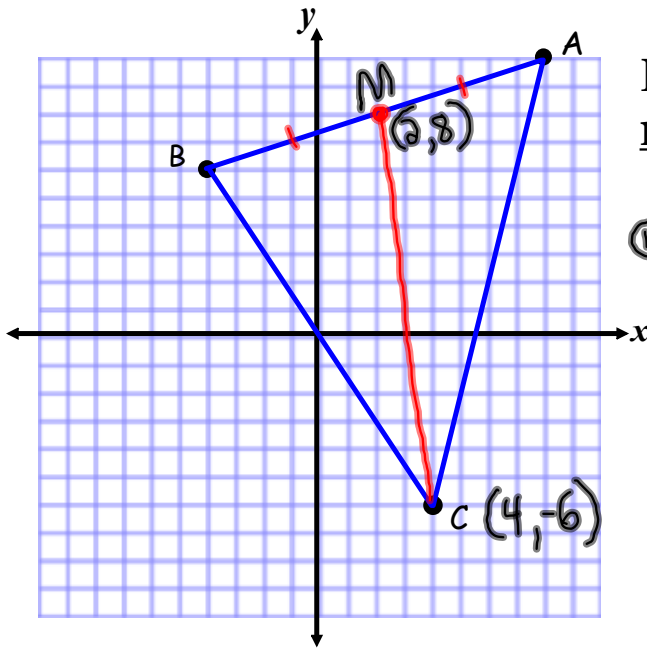
$$3y - 12 = -x - 6$$

$$x + 3y - 12 + 6 = 0$$

$$\boxed{x + 3y - 6 = 0}$$

Example:

ΔABC has vertices $A(8, 10)$; $B(-4, 6)$ & $C(4, -6)$.



Determine the equation of the median from C to AB.

$$\begin{aligned} \textcircled{1} M_{AB} &= \left[\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right] \\ &= \left[\frac{-4 + 8}{2}, \frac{6 + 10}{2} \right] \\ &= \left[\frac{4}{2}, \frac{16}{2} \right] \\ &= (2, 8) \end{aligned}$$

$$\begin{aligned} \textcircled{2} m_{cm} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8 - (-6)}{2 - 4} \\ &= \frac{14}{-2} \\ &= -7 \end{aligned}$$

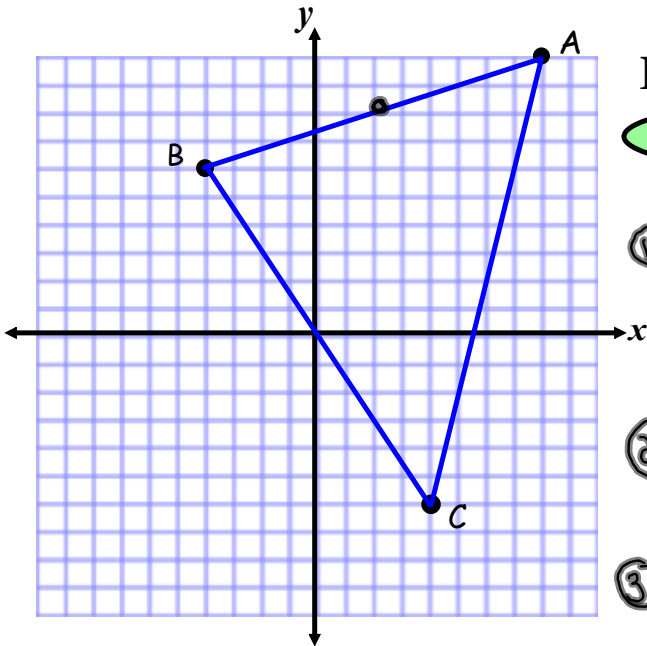
$$\begin{aligned} \textcircled{3} y - y_1 &= m(x - x_1) \\ y - 8 &= -7(x - 2) \\ y - 8 &= -7x + 14 \end{aligned}$$

$$7x + y - 8 - 14 = 0$$

$$\boxed{7x + y - 22 = 0}$$

Example:

ΔABC has vertices $A(8, 10)$; $B(-4, 6)$ & $C(4, -6)$.



Determine the equation of the
right bisector through AB

$$\textcircled{1} m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 10}{-4 - 8} = \frac{-4}{-12} = \frac{1}{3}$$

$$\textcircled{2} m_{\perp} = -3$$

$$\textcircled{3} M_{AB} = \left[\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right]$$

$$= \left[\frac{8 + (-4)}{2}, \frac{10 + 6}{2} \right]$$

$$= \left[\frac{4}{2}, \frac{16}{2} \right] = (2, 8)$$

$$\textcircled{4} y - y_1 = m(x - x_1)$$

$$y - 8 = -3(x - 2)$$

$$y - 8 = -3x + 6$$

$$3x + y - 8 - 6 = 0$$

$$\boxed{3x + y - 14 = 0}$$