

Arithmetic
(common difference "d")

$$t_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric
(Common Ratio "r")

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{a}{1 - r}$$

$$-1 < r < 1$$

$$\textcircled{1} \text{ a) } 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$$

$$a = 1$$

$$r = \frac{1}{3}$$

$$S_n = \frac{a}{1-r}$$

$$= \frac{1}{1 - \frac{1}{3}}$$

$$= \frac{1}{\frac{3}{3} - \frac{1}{3}}$$

$$= \frac{1}{\frac{2}{3}}$$

$$= 1 \times \frac{3}{2} = \boxed{\frac{3}{2}}$$

$$\text{c) } \frac{1}{4} - \frac{5}{16} + \frac{25}{64} - \frac{125}{256} + \dots$$

$$a = \frac{1}{4}$$

Diverging

$$r = -\frac{5}{4}$$

$$\lim_{n \rightarrow \infty} \left(\frac{1}{4}\right) \left(-\frac{5}{4}\right)^{n-1} = \text{DNE}$$

$$t_n = \left(\frac{1}{4}\right) \left(-\frac{5}{4}\right)^{n-1}$$

$$\textcircled{2} \text{ b) } \sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n$$

Series

$$\frac{-2}{5} + \frac{4}{25} - \frac{8}{125}$$

$$a = \frac{-2}{5}$$

$$r = -\frac{2}{5}$$

$$S_n = \frac{\frac{-2}{5}}{1 - \frac{2}{5}} = \frac{\frac{-2}{5}}{\frac{3}{5}} = \frac{-2}{5} \cdot \frac{5}{3}$$

$$= -\frac{10}{35}$$

$$= \boxed{\frac{-2}{7}}$$

$$\textcircled{5} \text{ c) } 4 + 8 + 12 + \dots + 400$$

$$a = 4$$

$$d = 4$$

$$t_n = 400$$

$$S_n = ?$$

Solve for "n"

$$t_n = a + (n-1)d$$

$$400 = 4 + (n-1)(4)$$

$$400 = 4 + 4n - 4$$

$$400 = 4n$$

$$100 = n$$

$$S_n = \frac{n}{2} (a + t_n)$$

$$S_{100} = \frac{100}{2} (4 + 400)$$

$$S_{100} = 50(404)$$

$$S_{100} = 20200$$

$$\textcircled{5} \text{ a) } \sum_{n=1}^5 2n+1$$

$$= 3 + 5 + 7 + 9 + 11$$

$$= 35$$

$$\textcircled{7} \quad t_5 = 48$$

$$t_5 = ar^{5-1}$$

$$t_5 = ar^4$$

$$ar^4 = 48$$

$$t_8 = 384$$

$$t_8 = ar^{8-1}$$

$$t_8 = ar^7$$

$$ar^7 = 384$$

$$\frac{ar^7 = 384}{ar^4 = 48}$$

$$r^3 = 8$$

$$r = 2$$

$$ar^4 = 48$$

$$a(2)^4 = 48$$

$$16a = 48$$

$$a = 3$$

$$t_n = ar^{n-1}$$

$$t_n = (3)(2)^{n-1}$$