Warm Up

Which of the following is the inverse of the matrix $\begin{pmatrix} 4 & -3 \\ 2 & -2 \end{pmatrix}$?

[A] $\begin{pmatrix} -1 & \frac{3}{2} \\ -1 & 2 \end{pmatrix}$
[B] $\begin{pmatrix} 2 & -1 \\ \frac{3}{2} & -1 \end{pmatrix}$
[C] $\begin{pmatrix} 1 & -\frac{3}{2} \\ 1 & -2 \end{pmatrix}$
[D] $\begin{pmatrix} \frac{1}{7} & -\frac{3}{14} \\ \frac{1}{7} & -\frac{2}{7} \end{pmatrix}$

Which system of equations would you use to represent the cost of these two newspaper classified rates?
- The "Daily Gleaner" has a flat rate of $18 plus 20¢ per word
- The "Times-Transcript" has a flat rate of $25 plus 10¢ per word

(A) $\begin{array}{l} C-18=20w \\ C-25=10w \end{array}$
(B) $\begin{array}{l} 18+C=20w \\ 25+C=10w \end{array}$
(C) $\begin{array}{l} C=0.2w+18 \\ C=0.1w+25 \end{array}$
(D) $\begin{array}{l} C-18w=0.2 \\ C-25w=0.1 \end{array}$

$(4a)$ R_1 + $4R_2$ $\begin{bmatrix} 9 & -4 & | & 1 & 0 \\ -2 & 1 & | & 0 & 1 \end{bmatrix}$

$2R_1$ + $9R_2$

$$\begin{bmatrix} 1 & 0 & | & 1 & 4 \\ 0 & 1 & | & 2 & 9 \end{bmatrix}$$ ← Inverse

⑤ c) $3x - 2y + 5z = 1$

$4x + 5y - 3z = 17$

$7x - 3y + 2z = 36$

$\begin{bmatrix} 3 & -2 & 5 & | & 1 \\ 4 & 5 & -3 & | & 17 \\ 7 & -3 & 2 & | & 36 \end{bmatrix}$ $3R_2 - 4R_1$ $\begin{bmatrix} 3 & -2 & 5 & | & 1 \\ 0 & 23 & -29 & | & 47 \\ 0 & 5 & -29 & | & 101 \end{bmatrix}$ $23R_3 - 5R_2$ $\begin{bmatrix} 3 & -2 & 5 & | & 1 \\ 0 & 23 & -29 & | & 47 \\ 0 & 0 & -522 & | & 2088 \end{bmatrix}$

$3R_3 - 7R_1$

$-522z = 2088$ $23y - 29z = 47$ $3x - 2y + 5z = 1$

$\boxed{z = -4}$ $23y - 29(-4) = 47$ $3x - 2(-3) + 5(-4) = 1$

$23y + 116 = 47$ $3x + 6 - 20 = 1$

$23y = -69$ $3x - 14 = 1$

$\boxed{y = -3}$ $3x = 15$

$\boxed{x = 5}$

$\underline{(5, -3, -4)}$

BONUS!!

Determine a, b, and c so that the points (-1 , 5), (2 , -1), and (3 , 13) are on the graph of $f(x) = ax^2 + bx + c$.

x, y

$$y = ax^2 + bx + c$$

(-1, 5)

$5 = a(-1)^2 + b(-1) + c$

$5 = a - b + c$

$a - b + c = 5$

(2, -1)

$-1 = a(2)^2 + b(2) + c$

$-1 = 4a + 2b + c$

$4a + 2b + c = -1$

(3, 13)

$13 = a(3)^2 + b(3) + c$

$13 = 9a + 3b + c$

$9a + 3b + c = 13$

$a - b + c = 5$
$4a + 2b + c = -1$
$9a + 3b + c = 13$

$$\begin{bmatrix} 1 & -1 & 1 & | & 5 \\ 4 & 2 & 1 & | & -1 \\ 9 & 3 & 1 & | & 13 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 4 \\ 0 & 1 & 0 & | & -6 \\ 0 & 0 & 1 & | & -5 \end{bmatrix}$$

$a = 4$
$b = -6$
$c = -5$

① $\begin{bmatrix} 2x+y & 5 \\ -1 & y-x \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ -1 & 1 \end{bmatrix}$

$2x+y = -2$ $2x+y = -2$ $2x+y = -2$

$y-x = 1$ $(-)$ $-x+y = 1$ $2(-1)+y = -2$

$\overline{}$

$3x = -3$ $-2+y = -2$

$\boxed{x = -1}$ $\boxed{y = 0}$

④ a) $2x + 3y + 7z = 15$ $10x + 15y + 35z = 75$ $10x + 8y - 8z = -4$
   $5x + 4y - 4z = -2$ (-) $10x + 8y - 8z = -4$ (+)$-10x + 5y + 10z = -5$
   $-2x + y + 2z = -1$ ———————————————— ————————————
                             $7y + 43z = 79$ $13y + 2z = -9$

$91y + 559z = 1027$ → $13y + 2z = -9$ ⌐ $2x + 3y + 7z = 15$
(-) $91y + 14z = -63$ $13y + 2(2) = 9$ $2x + 3(-1) + 7(2) = 15$
———————————————— $13y + 4 = -9$ $2x - 3 + 14 = 15$
        $545z = 1090$ $13y = -13$ $2x + 11 = 15$
        $z = 2$ $y = -1$ $2x = 4$
                                               $x = 2$

$(2, -1, 2)$

④ a) $2x + 3y + 7z = 15$ $\begin{bmatrix} 2 & 3 & 7 & | & 15 \\ 5 & 4 & -4 & | & -2 \\ -2 & 1 & 2 & | & -1 \end{bmatrix}$
   $5x + 4y - 4z = -2$
   $-2x + y + 2z = -1$

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rref([A]
  [[1 0 0 2]
   [0 1 0 -1]
   [0 0 1 2]]
■
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$(2, -1, 2)$

⑤ a) $3x - 4y + 5z = 26$

$6x - 2y - 3z = -39$

$x + 3y - 4z = -31$

$$\begin{bmatrix} 3 & -4 & 5 & | & 26 \\ 6 & -2 & -3 & | & -39 \\ 1 & 3 & -4 & | & -31 \end{bmatrix} \begin{array}{l} 2R1 - R2 \\ 3R3 - R1 \end{array} \begin{bmatrix} 3 & -4 & 5 & | & 26 \\ 0 & -6 & 13 & | & 91 \\ 0 & 13 & -17 & | & -119 \end{bmatrix} \begin{array}{l} 13R2 + 6R3 \end{array} \begin{bmatrix} 3 & -4 & 5 & | & 26 \\ 0 & -6 & 13 & | & 91 \\ 0 & 0 & 67 & | & 469 \end{bmatrix}$$

$67z = 469$

$\boxed{z = 7}$

$-6y + 13z = 91$

$-6y + 13(7) = 91$

$-6y + 91 = 91$

$-6y = 0$

$\boxed{y = 0}$

$3x - 4y + 5z = 26$

$3x - 4(0) + 5(7) = 26$

$3x + 35 = 26$

$3x = -9$

$\boxed{x = -3}$

$(-3, 0, 7)$

⑤ a) $3x - 4y + 5z = 26$

$6x - 2y - 3z = -39$

$x + 3y - 4z = -31$

$$\begin{bmatrix} 3 & -4 & 5 & | & 26 \\ 6 & -2 & -3 & | & -39 \\ 1 & 3 & -4 & | & -31 \end{bmatrix}$$

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rref([B]
    [[1 0 0 -3]
     [0 1 0 0 ]
     [0 0 1 7 ]]
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$(-3, 0, 7)$

Review Sheet Homework

6, 8, 10, 11, 12, 20, 21

⑧ Let x = investment @ 8%

Let y = " @ 10%

Let z = " @ 16%

$$x + y + z = 9000$$

$$0.08x + 0.10y + 0.16z = 1160$$

$$-0.08x - 0.10y + 0.16z = 440$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 9000 \\ 0.08 & 0.1 & 0.16 & | & 1160 \\ -0.08 & -0.1 & 0.16 & | & 440 \end{bmatrix}$$

```
rref([A]
  [[1 0 0 2000]
   [0 1 0 2000]
   [0 0 1 5000]]
■
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④ c) $x + y - z = 1$

$\boxed{2y + z = 1}$

$3x - y - 5z = 1$

$3x + 3y - 3z = 3$

$(-) \; 3x - y - 5z = 1$

$\overline{\boxed{4y + 2z = 2}}$

$4y + 2z = 2$

$(-) \; 4y + 2z = 2$

$\overline{\boxed{0 = 0}}$

let $\boxed{z = t} \rightarrow$ $2y + z = 1$

$2y + t = 1$

$2y = 1 - t$

$\boxed{y = \dfrac{1-t}{2}}$

$x + y - z = 1$

$x + \dfrac{1-t}{2} - t = 1$

$x + \dfrac{1}{2} - \dfrac{t}{2} - \dfrac{2t}{2} = 1$

$x + \dfrac{1}{2} - \dfrac{3t}{2} = 1$

$x = 1 - \dfrac{1}{2} + \dfrac{3t}{2}$

$\boxed{x = \dfrac{1+3t}{2}}$

$\left(\dfrac{1+3t}{2}, \; \dfrac{1-t}{2}, \; t \right)$

9

④ b) $x + y - z = 1$ $3x + 3y - 3z = 3$

    $\boxed{2y + z = 3}$ $\Leftrightarrow$ $\underline{3x - y - 5z = 0}$

    $3x - y - 5z = 0$ $\boxed{4y + 2z = 3}$

   $4y + 2z = 6$ **No Solution**

  $\Leftrightarrow$ $\underline{4y + 2z = 3}$

    $\boxed{0 = 3}$

6a) $3x - 4y + 5z = 26$

$6x - 2y - 3z = -39$

$x + 3y - 4z = -31$

$$\begin{bmatrix} 3 & -4 & 5 & | & 26 \\ 6 & -2 & -3 & | & -39 \\ 1 & 3 & -4 & | & -31 \end{bmatrix} \xrightarrow{\substack{R2 - 2R1 \\ 3R3 - R1}} \begin{bmatrix} 3 & -4 & 5 & | & 26 \\ 0 & 6 & -13 & | & -91 \\ 0 & 13 & -17 & | & -119 \end{bmatrix} \xrightarrow{6R3 - 13R2} \begin{bmatrix} 3 & -4 & 5 & | & 26 \\ 0 & 6 & -13 & | & -91 \\ 0 & 0 & 67 & | & 469 \end{bmatrix}$$

$67z = 469$

$\boxed{z = 7}$

$6y - 13z = -91$

$6y - 13(7) = -91$

$6y - 91 = -91$

$6y = 0$

$\boxed{y = 0}$

$3x - 4y + 5z = 26$

$3x - 4y + 5z = 26$

$3x - 4(0) + 5(7) = 26$

$3x - 0 + 35 = 26$

$3x = -9$

$\boxed{x = -3}$

11