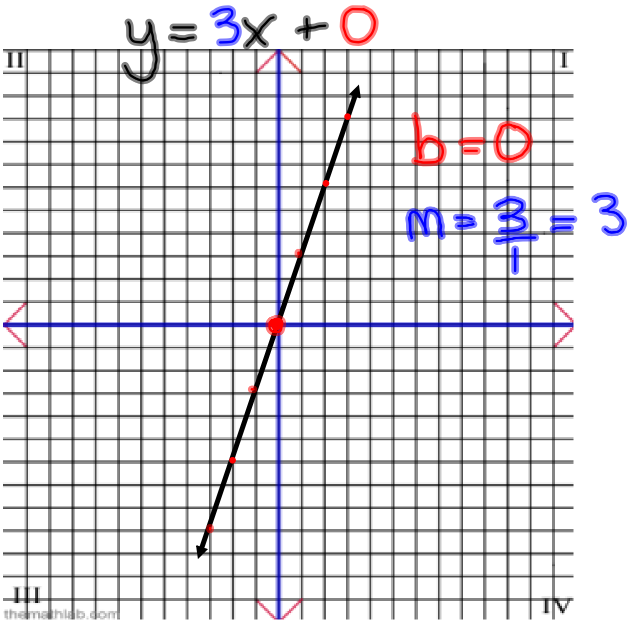
Linear Equations: $y = mx + b$

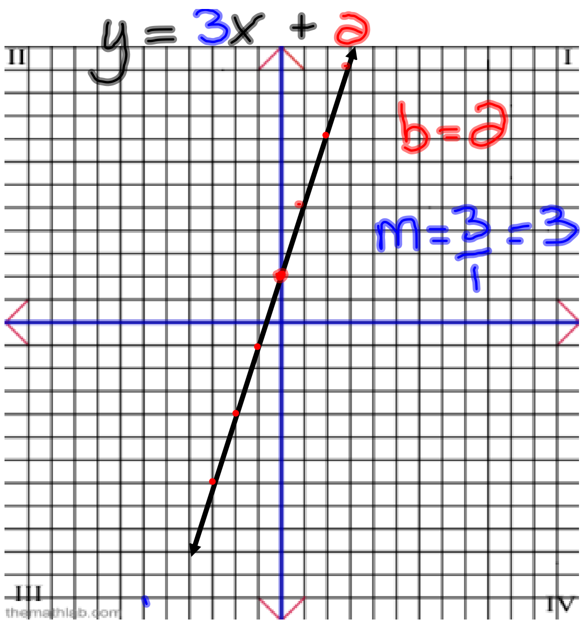
Slope

point
y - int

A linear relation can be defined by its slope and any point on the line.

Examine the graphs of the linear relations shown on the next page.



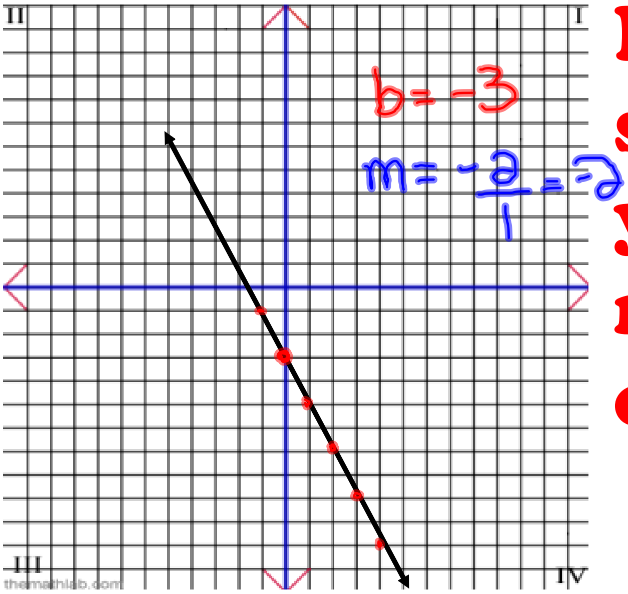


slope of 3 y-intercept of 0

Equation: y = 3x

slope of 3 y-intercept of 2

Equation: 3x + 2



Notice how the slope and the y-intercept relate to the equation.

slope of -2 y-intercept of -3

Equation: y = -2x - 3

Any linear relation can be expressed as $y = mx + b$, where m is the slope of the relation and b is the y-intercept.

This is called the slope y-intercept form of an equation.

Example 1:

Determine the slope and y-intercept of the line given by $3x - 4y = 12$.

Solution:

$$3x - 4y = 12$$

$$-4y = -3x + 12$$

$$y = \boxed{\frac{3}{4}}x \boxed{-3}$$

Thus, the slope is ¾ and the y-intercept is -3.

Example 2:

The lines represented by $y + 2 = 2(x - 3) + kx$ and $3(x + 2) = 3 + y$ have equal slopes. Find the value of "k".

Solution:

Write each equation in the slope y-intercept form and compare the slopes.

$y + 2 = 2(x-3) + kx$	$3(x+2) = 3 + y$
$y + 2 = 2x - 6 + kx$	$3x + 6 = 3 + y$
$y = (2x + kx) - 8$	$3x + 3 = y$
factor $y = (2 + k)x - 8$	$y = 3x + 3$
$m = (2+k)$ $b = -8$	$m = 3$ $b = 3$

If the slopes are equal then, $2 + k = 3$

$k = 3 - 2$

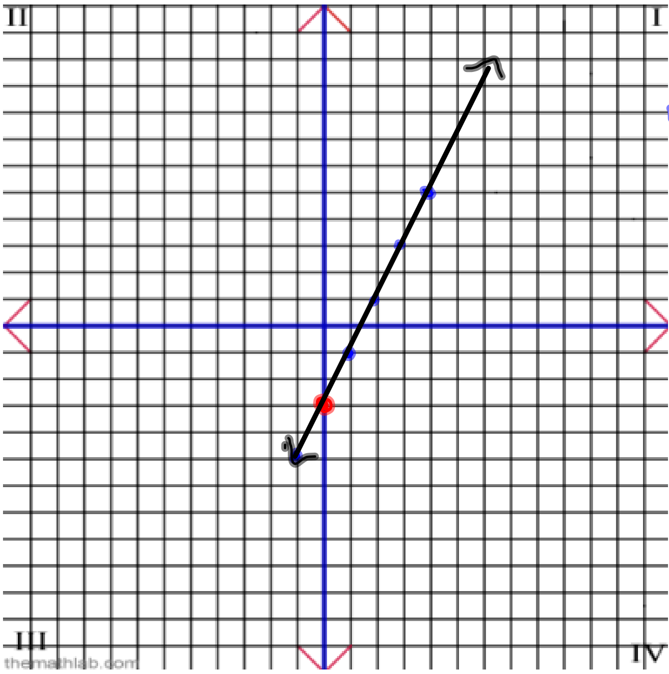
$k = 1$

Example 3:

Sketch the line which has: m = 2, b = -3

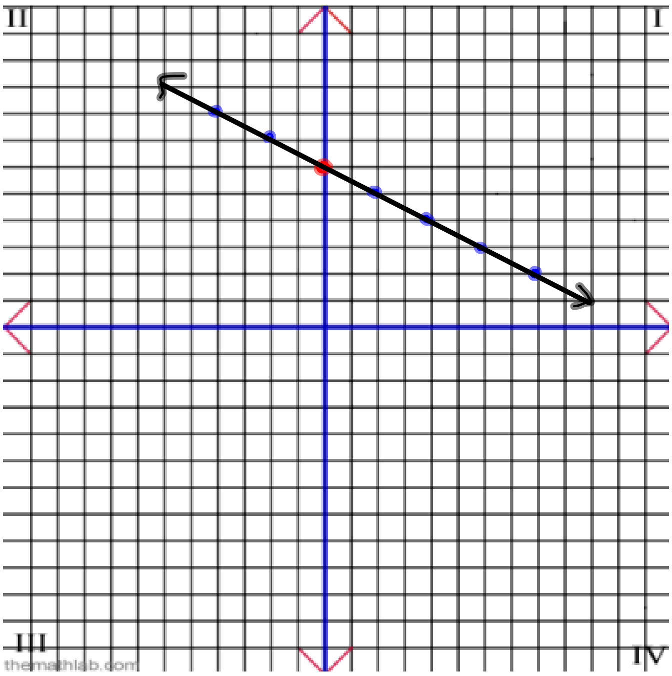
y int (0,-3)

$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1}$



Example 4:

Sketch the line which has: m = $-\frac{1}{2}$, b = 6



y int: (0,6)

$m = \frac{rise}{run} = \frac{-1}{2}$

8. $A(-1,1)$; $B(1, p+4)$; $C(-1-p, -4)$

If these points are collinear, $m_{AB} = m_{BC}$.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(p+4)-1}{1--1} = \frac{p+4-1}{2} = \frac{p+3}{2}$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(p+4)}{(-1-p)-1} = \frac{-4-p-4}{-1-p-1} = \frac{-8-p}{-2-p}$$

Since $m_{AB} = m_{BC}$: $\frac{p+3}{2} = \frac{-8-p}{-2-p}$

$$(p+3)(-2-p) = (2)(-8-p)$$
$$-2p - p^2 - 6 - 3p = -16 - 2p$$
$$-p^2 - 5p - 6 = -16 - 2p$$
$$-p^2 - 5p + 2p - 6 + 16 = 0$$
$$-p^2 - 3p + 10 = 0$$

OR

$$0 = p^2 + 3p - 10$$
$$0 = (p+5)(p-2)$$

Either:

$p+5=0$ or $p-2=0$

$p=-5$ or $p=2$