Parallel & Perpendicular Lines
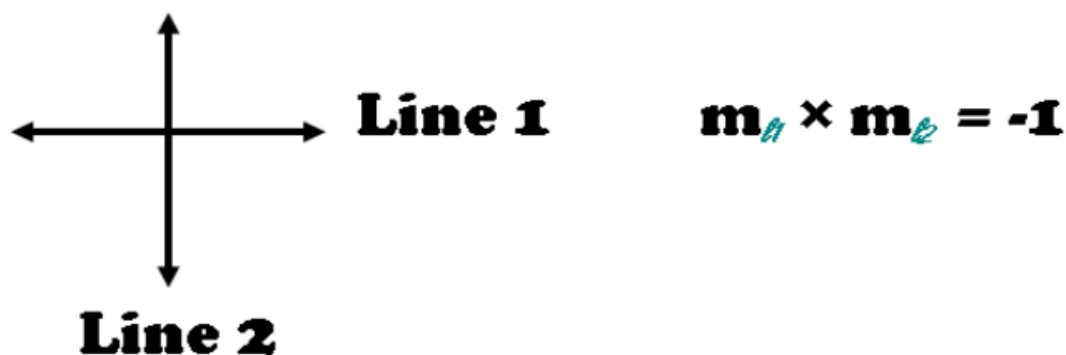
Parallel Lines

Two lines are parallel if they have the same slope.

Line 1 (ℓ_1)　　　Line 2 (ℓ_2)

$$m_{\ell_1} = m_{\ell_2}$$
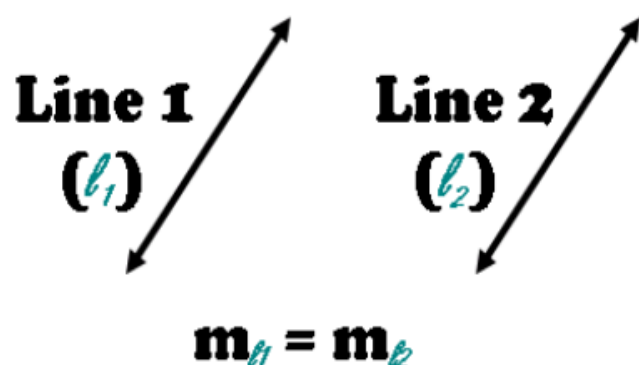
Perpendicular Lines

Two lines are perpendicular if the product of their slopes is -1. In other words, the slopes of the lines are negative reciprocals of each other.

Line 1

Line 2

$$m_{l1} \times m_{l2} = -1$$

Example 1

Show that the line through A(0 , 3) and B(1 , 5) is parallel to the line through C(1 , 4) and D(2 , 6).

Solution:

Find the slope of each pair of points.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 3}{1 - 0} \qquad\qquad = \frac{6 - 4}{2 - 1}$$

$$= \frac{2}{1} \qquad\qquad\quad = \frac{2}{1}$$

$$= 2 \qquad\qquad\qquad = 2$$

$m_{AB} = m_{CD}$, **therefore these two lines are parallel.**

Example 2

Show that the line through A(-1 , -2) and B(-3 , -5) is perpendicular to the line through C(1 , 0) and D(4 , -2).

Solution:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - -2}{-3 - -1} \qquad\qquad = \frac{-2 - 0}{4 - 1}$$

$$= \frac{-3}{-2} \qquad\qquad\quad = \frac{-2}{3}$$

$$= \frac{3}{2}$$

m_{AB} is the negative reciprocal of m_{CD}, therefore the two lines are perpendicular.

Example 3

The vertices of △ABC are A(-3, 2), B(2, 3) and C(3, -2). Determine whether △ABC is a right triangle.

Solution

Calculate the slope of each side of ▲ ABC.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - 2}{2 - {}^-3}$$

$$= \frac{1}{5}$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{{}^-2 - 3}{3 - 2}$$

$$= \frac{{}^-5}{1}$$

$$= {}^-5$$

$$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{{}^-2 - 2}{3 - {}^-3}$$

$$= \frac{{}^-4}{6}$$

$$= \frac{{}^-2}{3}$$

Since m_{AB} is the negative reciprocal of m_{BC}, we know that AB and BC are perpendicular to each other.

△ ABC is therefore a right triangle.