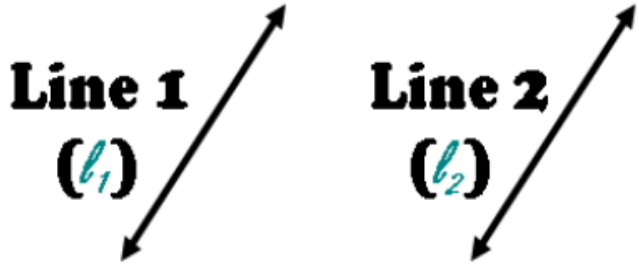
Parallel & Perpendicular Lines

Parallel Lines

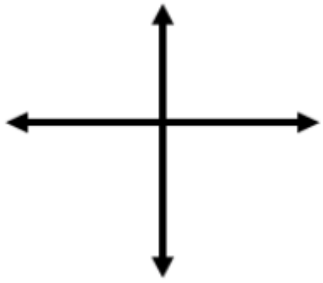
Two lines are parallel if they have the same slope.



$m_{\ell_1} = m_{\ell_2}$

Perpendicular Lines

Two lines are perpendicular if the product of their slopes is -1. In other words, the slopes of the lines are negative reciprocals of each other.



Line 1

Line 2

$m_{l_1} \times m_{l_2} = -1$

$m = \frac{3}{4}$

$\perp m = -\frac{4}{3}$

Example 1

Show that the line through A(0 , 3) and B(1 , 5) is parallel to the line through C(1 , 4) and D(2 , 6).

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-3}{1-0}$$

$$= \frac{2}{1}$$

$$= 2$$

$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-4}{2-1}$$

$$= \frac{2}{1}$$

$$= 2$$

Solution:

Find the slope of each pair of points.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{1 - 0} = \frac{2}{1} = 2$$

$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 4}{2 - 1} = \frac{2}{1} = 2$$

$m_{AB} = m_{CD}$, therefore these two lines are parallel.

Example 2

Show that the line through A(-1 , -2) and B(-3 , -5) is perpendicular to the line through C(1 , 0) and D(4 , -2).

Solution:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - -2}{-3 - -1} = \frac{-3}{-2} = \frac{3}{2}$$

$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{4 - 1} = \frac{-2}{3}$$

m_{AB} is the negative reciprocal of m_{CD}, therefore the two lines are perpendicular.

Example 3

The vertices of $\triangle ABC$ are A(-3, 2), B(2, 3) and C(3, -2). Determine whether $\triangle ABC$ is a right triangle.

Solution

Calculate the slope of each side of ▲ ABC.

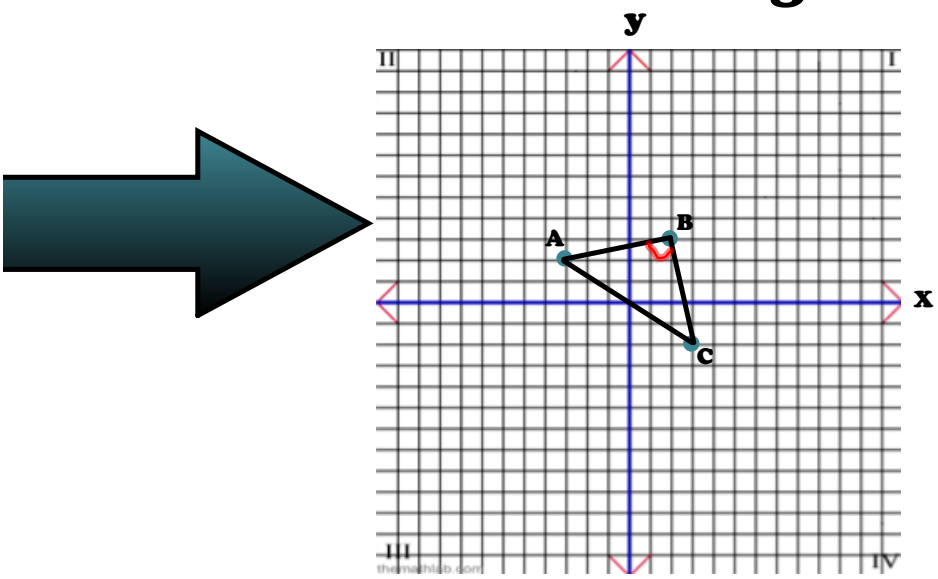
$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{2 - -3} = \frac{1}{5}$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{3 - 2} = \frac{-5}{1} = -5$$

$$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{3 - -3} = \frac{-4}{6} = \frac{-2}{3}$$

Since m_{AB} is the negative reciprocal of m_{BC}, we know that AB and BC are perpendicular to each other.

△ABC is therefore a right triangle.



③e) $y = 8$

$y = 0x + 8$

$m = \frac{0}{1}$ (Horizontal)

perp. $m = -\frac{1}{0}$ (Vertical)

③f) $x = -6$ (Vertical)

$m = \frac{1}{0}$

perp. $m = -\frac{0}{1}$