

Linear Equations: $Ax + By + C = 0$

You have seen that an equation of a linear relation can be written in the form $y = mx + b$.

In this section, you will be introduced to the **GENERAL form of a linear relation, $Ax + By + C = 0$.**

Example 1

Write $y = 3x + 2$ in the form $Ax + By + C = 0$.

Solution

$$y = 3x + 2$$

$$0 = 3x - y + 2 \text{ or } 3x - y + 2 = 0$$

We know that equations of lines can be found from information about the line.

You also know that when you are given the slope and the y-intercept that you can use $y = mx + b$ to determine the equation of a line.

What if you are given any two points and are asked to determine the equation of a line?

We can rearrange the slope formula to

get: $y - \underline{y_1} = m (x - \underline{x_1})$

↑
Slope

(x_1, y_1)
Point

* Need
• Slope
• Point

You now can use the two points that you are given to determine the slope, and then substitute the slope and one of the points (your choice) into the above equation.

Example 2

Find the equation of the line passing through (1, 1) and (6, 4).

Solution:

Step 1: Find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 1}{6 - 1} \\ &= \frac{3}{5} \end{aligned}$$

Step 2: Substitute the slope and one point into $y - y_1 = m(x - x_1)$

Point $(x_1, y_1) \Rightarrow (6, 4)$

$$m = \frac{3}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{3}{5}(x - 6)$$

$$y - 4 = \frac{3x}{5} - \frac{18}{5}$$

To remove the fractions, multiply by 5.

$$5y - 20 = 3x - 18$$

$$-3x + 5y - 2 = 0$$

$$3x - 5y + 2 = 0$$

Rearrange into GENERAL form:

$$5y = 3x - 18 + 20$$

$$5y = 3x + 2$$

$$0 = 3x - 5y + 2 \text{ or } 3x - 5y + 2 = 0$$

