Coordinates of the Centre and Radius

The **general form** of a circle is given as:

Standard

general

$$Ax^2 + By^2 + Cx + Dy + E = 0 \rightarrow (x-h)^2 + (y-k)^2 = r^2$$

When a circle is given in **general form** we must convert it into **standard form** to obtain the coordinates of the center and the length of the radius.

Remember that in **standard form**, we can state the center and radius of our circle easily.

To convert the equation of a circle from **general form** into **standard form** we make use of a strategy called *completing the square*.

General Form to Standard Form

$Ax^2 + By^2 + Cx + Dy + E \longrightarrow (x-h)^2 + (y-k)^2 = r^2$

Why? So we can read the radius and center

<u>Completing the Square</u>

*Note: When ***completing the square***, the coefficients of the squared terms must be **1**.

(Number in front of x^2 and y^2 have to be 1 and on the test or exam this will always be the case)

If you are given the equation of a circle in **<u>general form</u>**, such as $\mathbf{x^2 + y^2 + 6x - 10y - 12 = 0}$, you can use the following steps to convert it into **<u>standard form</u>**.

$$\mathbf{x^2 + y^2 + 6x - 10y - 12 = 0} \quad \textbf{\textit{(general form)}}$$

Step 1: Separate the **x** and **y** values as shown: (Take constant to the other side)

$$(x^2 + 6x \quad) + (y^2 - 10y \quad) = 12$$

Step 2: Next, take ½ of the x-value, square it, and add it to the left-hand side of the equation.

6x half of 6 is 3 then $3^2 = 9$ add to both sides

To compensate for this amount, you also have to add the same amount to the right-hand side of the equation. Repeat with the y-value.

$-10y$ half is -5 then $(-5)^2 = 25$

This results in:

$$x^2 + 6x + 9 + y^2 - 10y + 25 = \underbrace{12 + 9 + 25}_{\text{add}}$$

$$(x^2 + 6x + \mathbf{9}) + (y^2 - 10y + \mathbf{25}) = \mathbf{46}$$

Step 3: The terms $\mathbf{x^2 + 6x + 9}$ can be ***factored*** as: $(x + 3)^2$

The terms $\mathbf{y^2 - 10y + 25}$ can be ***factored*** as: $(y - 5)^2$

Step 4: This leaves you with: $\mathbf{(x + 3)^2 + (y - 5)^2 = 46}$

We can now state that this circle is centered at **(**-3, 5**)** with a radius of $\sqrt{46}$

$r^2 = 46$

$r = \sqrt{46}$

$(x-h)^2 + (y-k)^2 = r^2$

change signs

Center (h, k)

Radius $= \sqrt{r^2}$

$x^2 - 10x + 25$

half of m.c

$(x-5)^2$

$x^2 - 14x + 49$

$(x-7)^2$

$x^2 - 12x + 36$

$(x-6)^2$

You try

Find the radius and center of the following circle
(Hint: general to standard)

$$x^2 + y^2 + 4x - 6y - 9 = 0$$

Step 1) move constant to other side

$$x^2 + y^2 + 4x - 6y = 9$$

Step 2) Put "x" beside each other (Put "y" beside each other)

$$(x^2 + 4x \quad) + (y^2 - 6y \quad) = 9$$

Step 3) Half and square the number infront of x (Add to both sides)
Repeat for "y"

$$(x^2 + 4x + \underline{4}) + (y^2 - 6y + \underline{9}) = 9 + \underline{4} + \underline{9}$$

half = 2
squ = 2^2
= 4

half = -3
squ = $(-3)^2$
= 9

Step 4) Factor both brackets ($x \pm$ half of middle)

$$(x+2)^2 + (y-3)^2 = 22$$

$$\text{Center} = \overset{h,\ k}{(-2, 3)}$$ signs switch

Radius = $\sqrt{22}$

Worksheet # 3 - Equations of Circles

All Questions

Not on exam or test

****Sometimes, an equation in general form does not result in a circle.**

Look at the following examples:

Example 1

Find the center and the radius of: $\mathbf{x^2 + y^2 - 4x - 2y + 5 = 0}$

Solution

Step 1: $x^2 - 4x \quad + y^2 - 2y \quad = -5$

Step 2: $x^2 - 4x + \underline{\mathbf{4}} + y^2 - 2y + \underline{\mathbf{1}} = -5 + \mathbf{4} + \mathbf{1}$

Step 3: $(x - 2)^2 + (y - 1)^2 = 0$

We can know state that the center of the circle is located at **(2, 1)** and that the radius, $\mathbf{r} = \sqrt{0} = \mathbf{0}$

***When the radius is 0, this indicates that the equation represents the point **(2, 1)** only and **not** a circle.

Example 2 Not on test or exam

Find the center and the radius of: $\mathbf{2x^2 + 2y^2 - 8x - 4y + 12 = 0}$

Solution

The coefficients of the squared terms are not 1!

We have to divide ***each and every term*** by the coefficient of the squared terms (in order to have a circle, both coefficients must be the same).

Extra Step: Divide each term by 2 $\mathbf{x^2 + y^2 - 4x - 2y + 6 = 0}$

Step 1: $x^2 - 4x \quad + y^2 - 2y \quad = -6$

Step 2: $x^2 - 4x + \underline{\mathbf{4}} \quad + y^2 - 2y + \underline{\mathbf{1}} \quad = -6 + \mathbf{4} + \mathbf{1}$

Step 3: $(x - 2)^2 + (y - 1)^2 = \mathbf{-1}$

***__We can STOP right here!!! Recall that to find the radius, we take the square root of the value on the right-hand side of the equation. If we try to take the square root of a negative number, we do not get a *real* value for the radius.__**