Questions From Homework

② Let $x =$ 1st number
Let $y =$ 2nd number

$x + y = 8$
$x = \boxed{8-y}$
$x = 8 - 2$
$x = 6$

$S = x^2 + y^3$
$S = (8-y)^2 + y^3$
$S = 64 - 16y + y^2 + y^3$
$S' = -16 + 2y + 3y^2$ ← decomp.
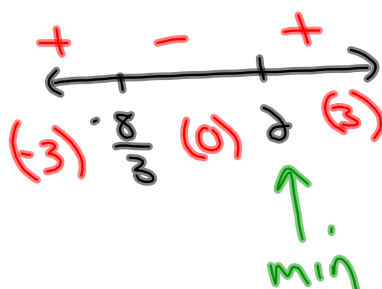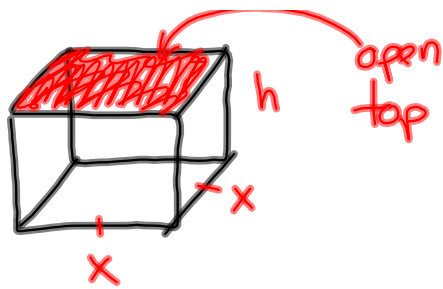$S' = 3y^2 + 2y - 16$
$S' = 3y^2 - 6y + 8y - 16$
$S' = 3y(y-2) + 8(y-2)$
$S' = (3y+8)(y-2)$

CV: $y = -\frac{8}{3} \boxed{2}$

$$\begin{array}{ccccc} + & & - & & + \\ \end{array}$$

$(-3) \quad -\frac{8}{3} \quad (0) \quad 2 \quad (3)$

↑ min

$A = x^2 + 4xh$

$A = x^2 + 4x\left[\dfrac{4000}{x^2}\right]$

$x^2 h = 4000$

$h = \boxed{\dfrac{4000}{x^2}}$

$A = x^2 + 16000x^{-1}$

$A' = 2x - \dfrac{16000}{x^2}$

$h = \dfrac{4000}{(20)^2}$

$A' = \dfrac{2x^3 - 16000}{x^2}$

$h = 10 \text{ cm}$

$2x^3 - 16000 = 0$

$2x^3 = 16000$

$x^3 = 8000$

$x = 20$
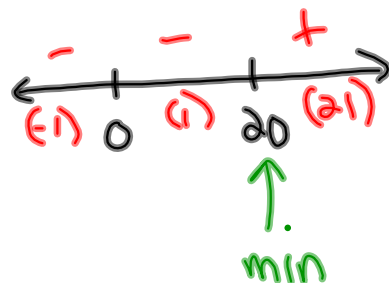
∴ The dimensions that minimize the surface area are 20 x 20 x 10

CV: $x = 0, \boxed{20}$

min

⑤ Find the point on the parabola $2y = x^2$ that is closest to the point (-4, 1)

$y = \frac{x^2}{2}$

$x_1 \qquad y_1$

$$d = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

$$d = \sqrt{(x+4)^2 + (y-1)^2}$$

$$d = \sqrt{(x+4)^2 + \left(\frac{1}{2}x^2 - 1\right)^2}$$

$$f(x) = (x+4)^2 + \left(\frac{1}{2}x^2 - 1\right)^2$$

$$f'(x) = 2(x+4)(1) + 2\left(\frac{1}{2}x^2 - 1\right)(x)$$

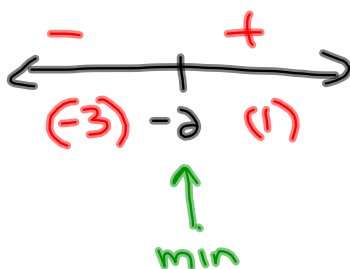$$f'(x) = 2x + 8 + x^3 - 2x$$

$$f'(x) = x^3 + 8$$

$x^3 = -8$

$\boxed{x = -2}$

$y = \frac{x^2}{2} \qquad (-2, 2)$

$y = \frac{(-2)^2}{2}$

$y = 2$

$\xleftarrow{\qquad - \qquad | \qquad + \qquad}$

(-3) -2 (1)

↑

min

You are in a dune buggy in the desert 12km due south of the nearest point A on a straight east-west road. You wish to get to point B on the road 10km east of point A. If your dune buggy can average 15km/h travelling over the desert, and 39km/h travelling on the road, toward what point on the road should you head to in order to minimize your travel time from A to B?

$$T = \frac{d}{s}$$

$$T = \frac{\sqrt{x^2+144}}{15} + \frac{10-x}{39}$$

$$T = \frac{1}{15}(x^2+144)^{1/2} + \frac{10}{39} - \frac{x}{39}$$
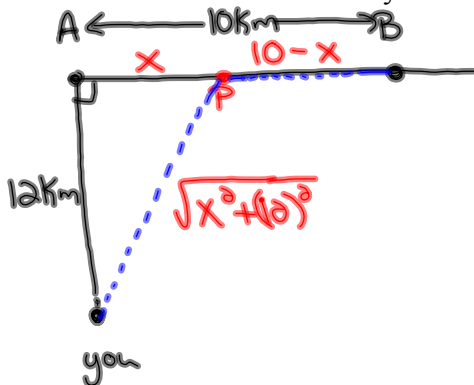
Let x = distance from A to P

$$T' = \frac{1}{30}(x^2+144)^{-1/2}(2x) + 0 - \frac{1}{39}$$

$$T' = \frac{x(x^2+144)^{-1/2}}{15} - \frac{1}{39}$$

$$0 = \frac{x}{15\sqrt{x^2+144}} - \frac{1}{39}$$

$$\frac{1}{39} = \frac{x}{15\sqrt{x^2+144}}$$

$$(39x)^2 = (15\sqrt{x^2+144})^2 \quad \text{square both}$$

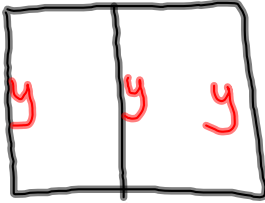$$1521x^2 = 225(x^2+144)$$

$$1521x^2 = 225x^2 + 32400$$

$$1296x^2 = 32400$$

$$x^2 = 25$$

$$x = \pm 5$$

∴ Head to a point 5km east of A

You have 400 m of fencing to construct a rectangular pen that will be divided into 2 sections of equal size. Find the dimensions that would maximize the area of the whole pen.

$$A = xy$$

Let x = length
Let y = width

$$A = x\left[\frac{400-2x}{3}\right]$$

$$A = \frac{400x - 2x^2}{3}$$

$$P = 2x + 3y$$

$$400 = 2x + 3y$$

$$A = \frac{400}{3}x - \frac{2}{3}x^2$$

$$400 - 2x = 3y$$

$$\boxed{\frac{400-2x}{3} = y}$$

$$A' = \frac{400}{3} - \frac{4}{3}x$$

$$0 = \frac{400}{3} - \frac{4}{3}x$$

$$\frac{4x}{3} = \frac{400}{3}$$

$$12x = 1200$$

$$x = 100 \text{ m}$$

$$y = \frac{400 - 2(100)}{3}$$

$$y = \frac{200}{3}$$

$$y = 66.\overline{6} \text{ m}$$

Find the points on the parabola $y = 6 - x^2$ that are closest to the point $(0, 3)$

Homework