SOLUTIONS ⇒ MAXIMUM/MINIMUM PROBLEMS

1. $h = -5t^2 + 50t + 40$

To find the solutions to "a" and "b" we will need to locate the vertex by completing the square.

① $h - 40 = -5t^2 + 50t$
② $h - 40 = -5(t^2 - 10t)$
③ $h - 40 - 125 = -5(t^2 - 10t + 25)$
④ $h - 165 = -5(t-5)^2$
⑤ $h = -5(t-5)^2 + 165$ (SF)

Vertex $(\underset{\text{Time}}{\underset{\uparrow}{5}}, \underset{\text{Height}}{\underset{\uparrow}{165}})$

a) It would take the object 5 seconds to reach its maximum height.

b) The maximum height reached by the object is 165 m.

① $h = -5t^2 + 50t + 40$

① $h - 40 = -5t^2 + 50t$

② $h - 40 = -5(t^2 - 10t)$

$-10 \times \frac{1}{2} = (-5)^2 = 25$

③ $h - 40 - 125 = -5(t^2 - \underline{10}t + 25)$

④ $h - 165 = -5(t-5)^2$

⑤ $h = -5(\underset{h}{\underset{\uparrow}{t-5}})^2 + \underset{k}{\underset{\uparrow}{165}}$ (Standard)

or

⑤ $-\frac{1}{5}(\underset{k}{\underset{\uparrow}{h-165}}) = -5(\underset{h}{\underset{\uparrow}{t-5}})^2$ (Transformational)

Vertex: $(h, k) = (5, 165)$

max height

Time to reach max height

a) 5 seconds

b) 165 m

2. $h = -4.9t^2 + 9.8t$

2 $h = -4.9(t^2 - 2t)$
3 $h - 4.9 = -4.9(t^2 - 2t + 1)$
4 $h - 4.9 = -4.9(t-1)^2$
5 $h = -4.9(t-1)^2 + 4.9$ (SF)

Vertex (1, 4.9)

Time   Height

a) The ball reaches a maximum height of 4.9 m after 1 second.

b) After 1 bounce $\Rightarrow$ 60% of 4.9 m
$= 0.60 \times 4.9$ m
$= 2.94$ m.

After 2 bounces $\Rightarrow$ 60% of 2.94 m
$= 0.60 \times 2.94$ m
$= 1.76$ m

Two bounces later, the ball will reach a maximum height of 1.76 m.

3. $h=-4.9t^2+19.6t$

(2) $h=-4.9(t^2-4t)$

(3) $h-19.6=-4.9(t^2-4t+4)$

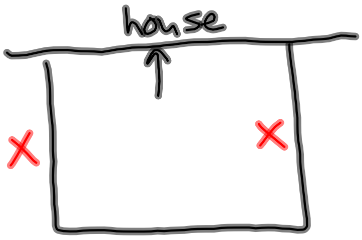
(4) $h-19.6=-4.9(t-2)^2$

(5) $h=-4.9(t-2)^2+19.6$ (SF)

Vertex $(2, 19.6)$

Time Height

The fire hydrant reaches a maximum height of 19.6m.



① $P = 24m$

② width $= x$

③ $\text{length} = \dfrac{P - \#\text{ of widths}}{\#\text{ of lengths}} = \dfrac{24 - 2x}{1} = 24 - 2x$

④ $A = x(24 - 2x)$

$A = 24x - 2x^2$

$A = -2x^2 + 24x$ (General)

$A = -2(x^2 - 12x)$

$A - 72 = -2(x^2 - 12x + 36)$

$A - 72 = -2(x-6)^2$

$A = -2(x-6)^2 + 72$

Vertex: $(6, 72)$

width max Area

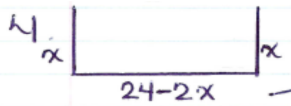
Length $= 24 - 2x$

$= 24 - 2(6)$

$= 24 - 12$

$= 12$

The dimensions are $12m \times 6m$



$P = 24m$

Let x = width

Then $24-2x$ = length

Area = length × width

$y = (24-2x)(x)$

↳ Place in General Form!

$y = 24x - 2x^2$ (Rearrange)

$y = -2x^2 + 24x$ (GF)

② $y = -2(x^2 - 12x)$

③ $y - 72 = -2(x^2 - 12x + 36)$

④ $y - 72 = -2(x-6)^2$

⑤ $y = -2(x-6)^2 + 72$ (SF)

Vertex $(6, 72)$

Width Area

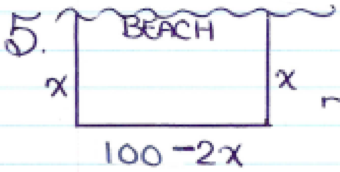
We now know that the maximum area of $72m^2$ occurs when the width is 6m.

To determine the length: $\frac{72m^2}{6m}$

$= 12m$.

Therefore, the dimensions of his flower garden will be $6m \times 12m$.

5.



$P = 100$ m

Let x = width

Then $100 - 2x$ = length

Area = length x width

$y = (100-2x)(x)$

↳ Place in General Form.

$y = 100x - 2x^2$ (Rearrange)

$y = -2x^2 + 100x$ (GF)

② $y = -2(x^2 - 50x)$

③ $y - 1250 = -2(x^2 - 50x + 625)$

④ $y - 1250 = -2(x-25)^2$

⑤ $y = -2(x-25)^2 + 1250$ (SF)

Vertex $(\underset{\text{Width}}{25}, \underset{\text{Area}}{1250})$

We now know that the maximum area of 1250 m² occurs when the width is 25m.

To determine the length: $\dfrac{1250\text{m}^2}{25\text{m}}$

$= 50$ m

=> Dimensions of swimming area will be 50m x 25m.

Review #1

① $(y+3) = (x-2)^2 \rightarrow$ Transformational

$y = (x-2)^2 - 3 \rightarrow$ Standard

Vertex: $(2, -3)$ Quadrant IV

② $\frac{1}{3}(y-2) = (x+3)^2 \rightarrow$ Transformational

Vertex: $(-3, 2)$

$y = 3(x+3)^2 + 2 \rightarrow$ Standard