② d) $y = \frac{(x-1)^3}{x^2} = \frac{x^3-3x^2+3x-1}{x^2}$

$\boxed{\text{Intercepts:}}$

xint (y=0)	yint (x=0)
$(x-1)^3=0$	$y=\frac{-1}{0}$ = undefined
$x-1=0$	No y intercept
$x=1$	
(1,0)	

$\boxed{\text{Asymptotes:}}$

V.A.

$x^2=0$

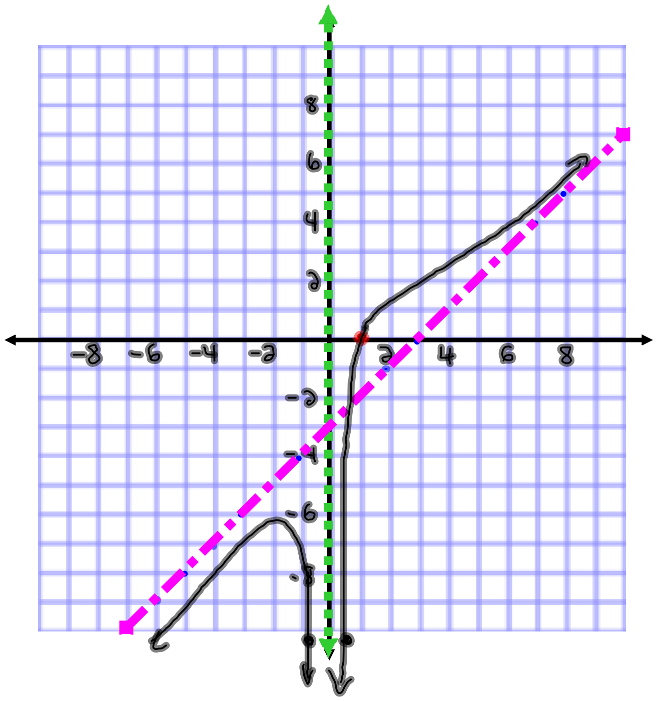
$x=0$

$$\begin{array}{r l} & x-3 \\ \underline{\underline{x^2}} \big) & \underline{x^3}-3x^2+3x-1 \\ & -x^3 \quad \downarrow \\ \hline & \underline{-3x^2}+3x-1 \\ & -3x^2 \\ \hline & 3x-1\,R \end{array}$$

S.A.

$y=x-3$

$m=\frac{1}{1}$ $b=-3$



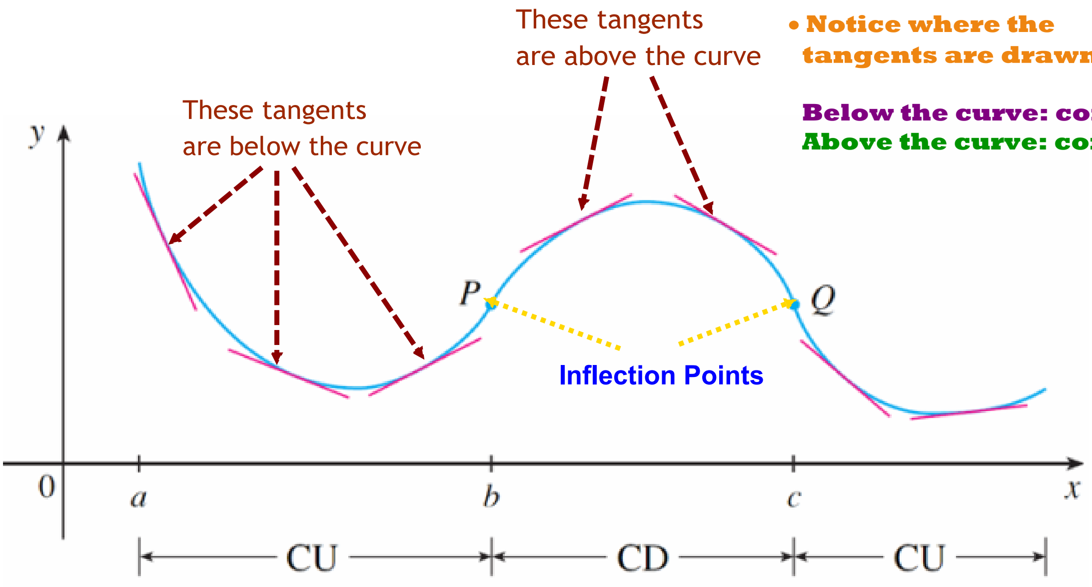
$\lim_{x\to0^-} \frac{(-)}{(+)} = -\infty$

$X=-0.01$

$\lim_{x\to0^+} \frac{(-)}{(+)} = -\infty$

$X=0.01$

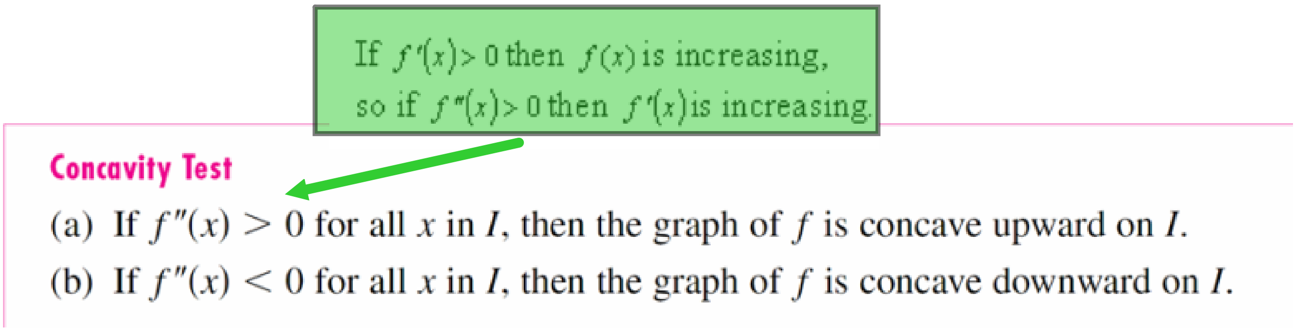
Concavity



- **Notice where the tangents are drawn...**

Below the curve: concave up
Above the curve: concave down

- In general, the graph of *f* is called ***concave upward*** on an interval *I* if it lies above all its tangents.
- It is called ***concave downward*** on *I* if it lies below all of these tangents.
- A point where a curve changes its direction of concavity is called an ***inflection point.***


If $f'(x) > 0$ then $f(x)$ is increasing,
so if $f''(x) > 0$ then $f'(x)$ is increasing.

Concavity Test

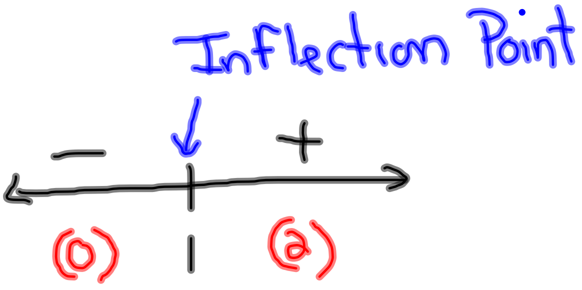
(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Thus there is a point of inflection at any point where the second derivative changes sign.

Determine where the curve $y = x^3 - 3x^2 + 4x - 5$
is concave upward and concave downward

Find the points of inflection



$y = x^3 - 3x^2 + 4x - 5$

$y' = 3x^2 - 6x + 4$

$y'' = 6x - 6$

$y'' = 6(x-1)$

$\boxed{\text{CV: } x = 1}$

Concave Up on $(1, \infty)$

Concave Down on $(-\infty, 1)$

Inflection Point: $(x=1)$

$y = x^3 - 3x^2 + 4x - 5$

$y = (1)^3 - 3(1)^2 + 4(1) - 5$

$y = 1 - 3 + 4 - 5$

$y = -3$

IP: $(1, -3)$

Determine where the curve $y = \dfrac{x}{x^2+1}$ is concave upward and concave downward

Find the points of inflection

$$y = \frac{x}{x^2+1}$$

$$y' = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \boxed{\frac{-x^2+1}{(x^2+1)^2}}$$

$$y'' = \frac{(x^2+1)^2(-2x) - (-x^2+1)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$y'' = \frac{-2x(x^2+1)^2 + 4x(x^2-1)(x^2+1)}{(x^2+1)^4}$$

$$y'' = \frac{2x\cancel{(x^2+1)}\left[\overset{-x^2-1}{-(x^2+1)}\overset{+2x^2-2}{+2(x^2-1)}\right]}{(x^2+1)^{\cancel{4}\,3}}$$

$$y'' = \frac{2x(x^2-3)}{(x^2+1)^3}$$ $\leftarrow$ Always positive

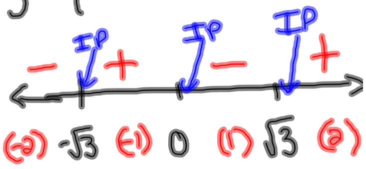
CV: $2x = 0$

$x = 0$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$



CU on $(-\sqrt{3},0)$ and $(\sqrt{3}$

CD on $(-\infty,-\sqrt{3})$ and $(0$

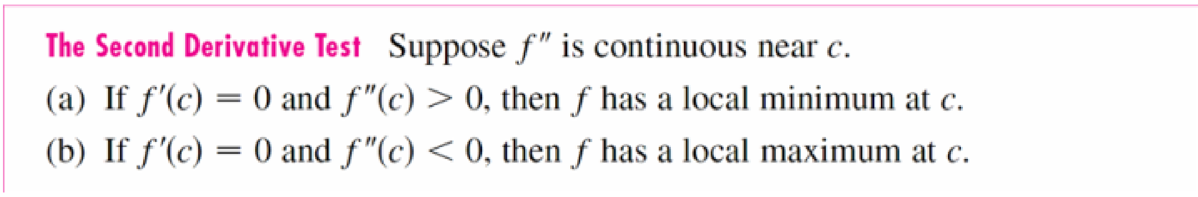
Inflection Points: $y = \dfrac{x}{x^2+1}$

$f(-\sqrt{3}) = \dfrac{-\sqrt{3}}{4}$ $\left(-\sqrt{3}, -\dfrac{\sqrt{3}}{4}\right)$

$F(0) = \dfrac{0}{1} = 0$ $(0,0)$

homework

Second Derivative Test for Local Extrema



The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

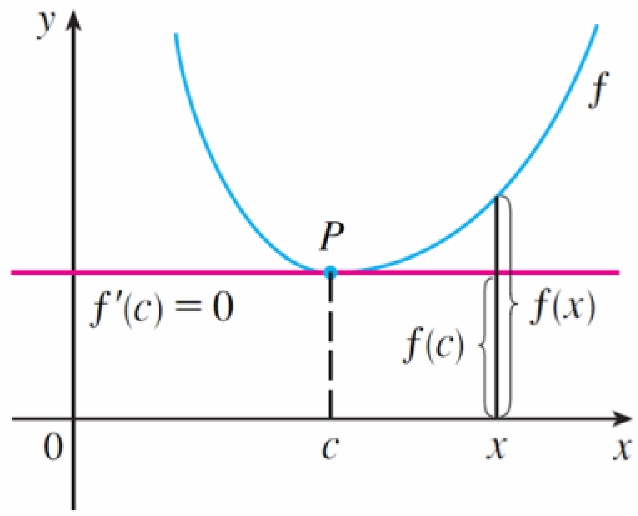
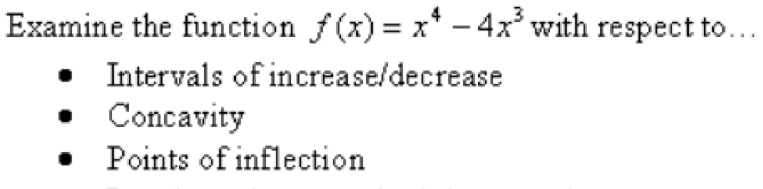


FIGURE 6

$f''(c) > 0$, f is concave upward

Example:



Examine the function $f(x) = x^4 - 4x^3$ with respect to…

- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values

Solution

Example:

Using the function: $f(x) = \frac{x^2}{x-7}$

Determine each of the following...

- Intercepts
- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values