

## Questions From Homework

② h)  $h(y) = \left(\frac{y}{3}\right)^2 = \frac{y^2}{9} = \frac{1}{9}y^2$

$h'(y) = \frac{2}{9}y$

j)  $f(x) = \sqrt[3]{x^2} = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} = \frac{2}{3\sqrt[3]{x}}$

③ e)  $y = \sqrt{x^3}$ ,  $x=8$

$y = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

$y'(8) = \frac{3\sqrt{8}}{2} = \frac{3\sqrt{4 \cdot 2}}{2} = \frac{6\sqrt{2}}{2} = \boxed{3\sqrt{2}}$

↑  
Slope of  
the  
tangent

④ c)  $xy = 1$   $x_1, y_1$   
 $(5, 1/5)$   
 $y = \frac{1}{x} = x^{-1}$

① Differentiate:

$y' = -x^{-2} = -\frac{1}{x^2}$

② Sub in x-value

$y'(5) = -\frac{1}{(5)^2}$

$y'(5) = \boxed{-\frac{1}{25}}$

Slope of  
the  
tangent  
"m".

③ Find Equation:

$y - y_1 = m(x - x_1)$

$y - \frac{1}{5} = -\frac{1}{25}(x - 5)$

$y - \frac{1}{5} = -\frac{x}{25} + \frac{5}{25}$

$25y - 5 = -x + 5$

$x + 25y - 10 = 0$

## Questions From Homework

$$\textcircled{5} \quad f(x) = \frac{1}{x} \quad | \quad f(x+h) = \frac{1}{x+h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \quad \text{multiply by: } (x)(x+h)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x - (x+h)}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x - x - h}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-h}{h(x)(x+h)} = \frac{-1}{x^2}$$

### Example:

Find the slope of the tangent line to the graph of the given function at the given x value.

$$g(x) = \sqrt[5]{x} \quad x = 32$$

$$g(x) = x^{1/5}$$

① Differentiate:

$$g'(x) = \frac{1}{5} x^{-4/5} = \frac{1}{5 x^{4/5}} = \frac{1}{5 \sqrt[5]{x^4}}$$

② Sub in x-value

$$g'(32) = \frac{1}{5(32)^{4/5}} = \frac{1}{5(16)} = \boxed{\frac{1}{80}}$$

← Slope of  
the tangent  
"m"

## Example:

Find the equation of the tangent line to the curve  $f(x) = x^6$  at the point  $(-2, 64)$

Remember that the equation of a line is found by using the point-slope formula...  $y - y_1 = m(x - x_1)$

The curve is the graph of the function  $f(x) = x^6$  and we know that the slope of the tangent line at  $(-2, 64)$  is the derivative  $f'(-2)$

- Find derivative
- Fill in x value and solve for slope
- Use equation of a line formula and solve

① Differentiate:

$$f(x) = x^6$$
$$f'(x) = 6x^5$$

② Sub in x-value:

$$f'(-2) = 6(-2)^5$$
$$= 6(-32)$$
$$= -192$$

← "m"

③ Find Equation:

$$y - y_1 = m(x - x_1)$$
$$y - 64 = -192(x + 2)$$

$$y - 64 = -192x - 384$$

$$192x + y + 320 = 0$$

## Sums and Differences

- These next rules say that the derivative of a sum (difference) of functions is the sum (difference) of the derivatives:

**The Sum Rule** If  $f$  and  $g$  are both differentiable, then

$$\frac{d}{dx} [f(x) + g(x)] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

**The Difference Rule** If  $f$  and  $g$  are both differentiable, then

$$\frac{d}{dx} [f(x) - g(x)] = \frac{d}{dx} f(x) - \frac{d}{dx} g(x)$$

Demonstrate what this all means...

Differentiate each of the following:

$$1. f(x) = 2x^4 + \sqrt{x} \\ = 2x^4 + x^{1/2}$$

$$f'(x) = 8x^3 + \frac{1}{2}x^{-1/2} \\ = 8x^3 + \frac{1}{2x^{1/2}}$$

$$2. f(x) = 6x^4 - 5x^3 - 2x + 17$$

$$f'(x) = 24x^3 - 15x^2 - 2x^0 + 0 \\ = 24x^3 - 15x^2 - 2$$

$$3. f(x) = (2x^3 - 5)^2 \\ = (2x^3 - 5)(2x^3 - 5) \\ = 4x^6 - 10x^3 - 10x^3 + 25 \\ = 4x^6 - 20x^3 + 25$$

$$f'(x) = 24x^5 - 60x^2$$

$$\textcircled{4} g(x) = 4x^3 - \frac{6}{x^2} + \sqrt[3]{x} \\ = 4x^3 - 6x^{-2} + x^{1/3}$$

$$g'(x) = 12x^2 + 12x^{-3} + \frac{1}{3}x^{-2/3} \\ = 12x^2 + \frac{12}{x^3} + \frac{1}{3x^{2/3}}$$

# Homework

$$\textcircled{1} \quad g) \quad y = \frac{x+1}{\sqrt{x}} = \frac{x+1}{x^{1/2}} = x^{-1/2} (x+1) = x^{1/2} + x^{-1/2}$$

$$y' = \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{-3/2}$$