METHOD #3

THE QUADRATIC FORMULA

We can use the method of completing the square to come up with a formula that can be used to solve **ALL** quadratic equations.

The solution to any quadratic equation: $ax^2 + bx + c = 0$; where $a \neq 0$, is given by:

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1:

Solve $\underline{1}x^2 + \underline{3}x - \underline{4} = 0$

Solution:

$\underline{a = 1}$; $\underline{b = 3}$; $\underline{c =-4}$

$$\text{Therefore, } x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9+16}}{2}$$

$$x = \frac{-3 \pm \sqrt{25}}{2}$$

$$x = \frac{-3 \pm 5}{2}$$

$$x = \frac{2}{2} \text{ or } x = \frac{-8}{2}$$

$$x = 1 \text{ or } x = -4$$

Example 2:

Solve $7x^2 - 4 = 0$

Solution: $\underline{7}x^2 + \underline{0}x - \underline{4} = 0$

$\underline{a = 7}$; $\underline{b = 0}$; $\underline{c =-4}$

$$\text{Therefore, } x = \frac{0 \pm \sqrt{(0)^2 - 4(7)(-4)}}{2(7)}$$

$$x = \frac{\pm\sqrt{0+112}}{14}$$

$$x = \frac{\pm\sqrt{112}}{14}$$

$$x = \frac{\pm\sqrt{112}}{14}$$

simplify

$$x = \frac{\pm\sqrt{16 \text{ x } 7}}{14}$$

$$x = \frac{\pm 4\sqrt{7}}{14}$$

Reduce

$$x = \frac{\pm 2\sqrt{7}}{7}$$

Solve by Factoring

$$x^2 + \overset{+}{3}x - \overset{\times}{4} = 0$$

$-1 \times 4 = -4$

$-1 + 4 = 3$

$$(x - 1)(x + 4) = 0$$

$$x \boxed{-1} = 0 \quad \Big| \quad x \boxed{+4} = 0$$

$$x = 1 \quad \Big| \quad x = -4$$

Since the solutions to quadratic equations are linked to the x-intercepts of quadratic functions, it makes sense that quadratic equations may also have 0, 1, or 2 solutions.

In the next few examples, we will use the quadratic formula to find the solution to various quadratic equations. These examples will illustrate the three possible results that can be obtained when solving quadratics.

Example 3: Two REAL Solutions

by factoring

Solve $x^2 + 7x + 12 = 0$

$(x+3)(x+4)=0$

Solution:

$a = 1$; $b = 7$; $c = 12$

Therefore, $x = \frac{-7\pm\sqrt{(7)^2-4(1)(12)}}{2(1)}$

$$x = \frac{-7\pm\sqrt{49-48}}{2}$$

$$x = \frac{-7\pm\sqrt{1}}{2}$$

$$x = \frac{-7\pm1}{2}$$

$$x = \frac{-6}{2} \text{ or } x = \frac{-8}{2}$$

$$x = -3 \text{ or } x = -4$$

Example 4: One REAL Solution

Solve $2x^2 + 24x + 72 = 0$

Solution:

$a = 2$; $b = 24$; $c = 72$

Therefore, $x = \frac{-24\pm\sqrt{(24)^2-4(2)(72)}}{2(2)}$

$$x = \frac{-24\pm\sqrt{576-576}}{4}$$

$$x = \frac{-24\pm\sqrt{0}}{4}$$

$$x = \frac{-24\pm0}{4}$$

$$x = \frac{-24}{4}$$

$$x = -6$$

Trinomial Decomposition →

$\underline{12} \times \underline{12} = 144$

$\underline{12} + \underline{12} = 24$

Solve by factoring:

$$\underline{\underline{2}}x^2+24x+\underline{\underline{72}}=0$$

$$(2x^2+12x)(+12x+72)=0$$

$$\underline{2x}\underline{(x+6)}+\underline{12}(x+6)=0$$

$$\underline{(x+6)}\underline{(2x+12)}=0$$

$x+6=0$ | $2x+12=0$

$x=-6$ | $2x=-12$

$x=-6$

Example 5: No REAL Solutions.

Solve $x^2 - 4x + 8 = 0$

Solution:

$a = 1$; $b = -4$; $c = 8$

Therefore, $x = \frac{-(-4)\pm\sqrt{(-4)^2-4(1)(8)}}{2(1)}$

$$x = \frac{4\pm\sqrt{16-32}}{2}$$

$$x = \frac{4\pm\sqrt{-16}}{2}$$

We will come back to this ...