

Solve for x:

$$x^2 + 5 = -6x$$

$$x^2 + 6x + 5 = 0$$

Method 1: Factoring

$$x^2 + 6x + 5 = 0 \quad (\text{Simple Trinomial})$$

$$(x+1)(x+5) = 0 \quad \begin{array}{l} \frac{1}{1} \times \frac{5}{5} = 5 \\ \underline{1} + \underline{5} = 6 \end{array}$$

$$\begin{array}{l|l} x+1=0 & x+5=0 \\ \hline x=-1 & x=-5 \end{array}$$

Method 2: Completing the Square

$$x^2 + 6x + 5 = 0$$

$$x^2 + 6x = -5$$

$$x^2 + \underline{6}x + \underline{9} = -5 + 9 \quad 6 \times \frac{1}{2} = (6)^2 = 9$$

$$(x+3)^2 = 4 \quad \leftarrow \text{square root both sides}$$

$$x+3 = \pm 2$$

$$x = -3 \pm 2$$

$$\begin{array}{l|l} x = -3 - 2 & x = -3 + 2 \\ \hline x = -5 & x = -1 \end{array}$$

Method 3: (Quadratic Formula) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x^2 + 6x + 5 = 0$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(5)}}{2(1)}$$

$$a = 1$$

$$b = 6$$

$$c = 5$$

$$x = \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$x = \frac{-6 \pm \sqrt{16}}{2}$$

$$x = \frac{-6 \pm 4}{2}$$

$$x = \frac{-6-4}{2}$$

$$x = \frac{-10}{2}$$

$$x = -5$$

$$x = \frac{-6+4}{2}$$

$$x = \frac{-2}{2}$$

$$x = -1$$

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IMAGINARY NUMBERS

Recall the solution to example 5 on the previous page:

$$x = \frac{4 \pm \sqrt{-16}}{2}$$

You will notice that we have a negative number under the square root sign, and we know that there is no *real* solution to the square root of a negative value. Therefore, we must now discuss a special type of number that will enable us to solve this problem.

While it is true that $\sqrt{-16}$ does not have a solution that is a *real* number, $\sqrt{-16}$ most definitely is a valid mathematical number. The fact is that it happens to belong to another set of numbers called *imaginary* or *complex numbers*.

Imaginary or *complex numbers* can be represented by the expression:

$a + bi$, where a and b are real numbers and $i = \sqrt{-1}$ or $i^2 = -1$

To use this definition of i : $\sqrt{-25} = \sqrt{25i^2} = 5i$

$$\sqrt{-100} = \sqrt{100i^2} = 10i$$

$$\sqrt{-43} = \sqrt{43i^2} = \sqrt{43}i$$

So if we look back at Example 5:

$$\begin{aligned}x &= \frac{4 \pm \sqrt{-16}}{2} \\x &= \frac{4 \pm \sqrt{16i^2}}{2} \\x &= \frac{4 \pm 4i}{2} \\&= 2 \pm 2i\end{aligned}$$

So, one solution is $x = 2 + 2i$, and the other solution is $x = 2 - 2i$. We say that this equation has no real roots, but we still must find the complex roots.

Recall, that the graph of the corresponding quadratic function would not have any x-intercepts!

$$\sqrt{-64} = \sqrt{64i^2} = 8i$$

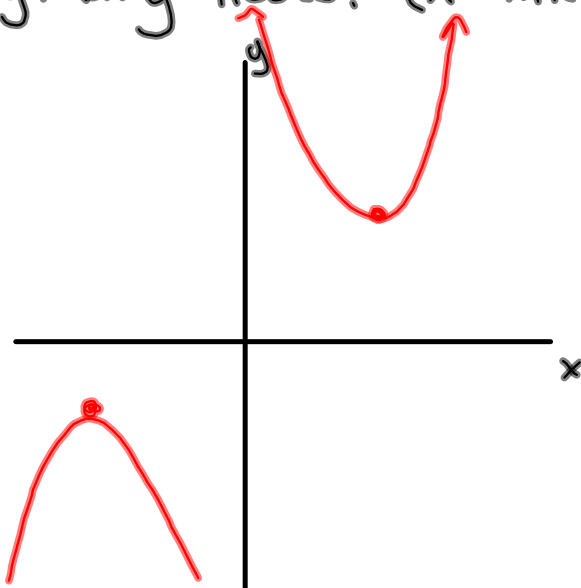
$$\sqrt{-49} = \sqrt{49i^2} = 7i$$

$$\sqrt{-25} = \sqrt{25i^2} = 5i$$

$$\begin{aligned}\sqrt{-12} &= \sqrt{12i^2} = i\sqrt{12} \\ &= i\sqrt{4 \times 3} \\ &= 2i\sqrt{3}\end{aligned}$$

Simplify

Imaginary Roots: (x-intercepts)



$$\textcircled{a} \quad x^2 + 5x + 8 = 0$$

$$a=1 \quad b=5 \quad c=8$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 32}}{2}$$

$$x = \frac{-5 \pm \sqrt{-7}}{2}$$

$$x = \frac{-5 \pm \sqrt{7i^2}}{2}$$

$$x = \frac{-5 \pm i\sqrt{7}}{2}$$