Math 11 **Predicting the Number and Type of Roots of Quadratic Equations**

We just stated that a quadratic equation can have 0, 1, or 2 solutions or **roots**. There is a simple way to predict the number and type of roots that an equation has. We use a quantity called the **Discriminant**, **D**. The ***discriminant*** is the quantity that is found under the square root sign in the quadratic formula:

$$\boxed{D = b^2 - 4ac}$$

For any quadratic equation written in General Form, $ax^2 + bx + c = 0$

If the discriminant, $D = b^2 - 4ac > 0$, there will be two **REAL** roots. → if D is positive

If the discriminant, $D = b^2 - 4ac = 0$, there will be one **REAL** root. → if $D = 0$

If the discriminant, $D = b^2 - 4ac < 0$, there will be two **IMAGINARY** roots. if D is negative
or 0 REAL roots.

Of course, we can extend this theory to the number of x-intercepts a quadratic function has. We can easily tell if a function has 0, 1, or 2 x-intercepts by evaluating the discriminant of the function and applying the same rules as above.

Example 1 $D = b^2 - 4ac$

Find the discriminant for each of the following quadratic equations. Then predict the type of roots.

a) $2x^2 + 3x - 2 = 0$

$a = 2 \quad b = 3 \quad c = -2$

Solution: $D = b^2 - 4ac$

$D = (3)^2 - 4(2)(-2)$

$D = 9 + 16$

$D = 25$ positive

Since $D > 0$, 2 REAL Roots.

b) $-3x^2 - 2x - 1 = 0$

$a = -3 \quad b = -2 \quad c = -1$

Solution: $D = b^2 - 4ac$

$D = (-2)^2 - 4(-3)(-1)$

$D = 4 - 12$

$D = -8$ negative

Since $D < 0$, 2 IMAGINARY Roots.

0 REAL Roots

c) $x^2 + 16x + 64 = 0$

$a = 1 \quad b = 16 \quad c = 64$

Solution: $D = b^2 - 4ac$

$D = (16)^2 - 4(1)(64)$

$D = 256 - 256$

$\boxed{D = 0}$

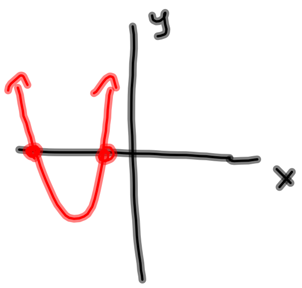
Since $D = 0$, 1 REAL Root.

Example 2

Find the number of x-intercepts, or zeros, that each of the corresponding quadratic functions from **Example 1** will have.

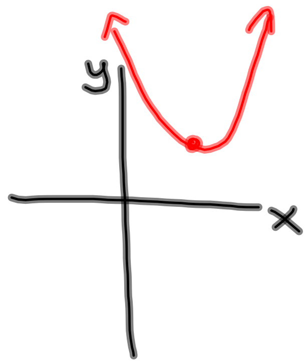
a) $y = 2x^2 + 3x - 2$

2 x-intercepts



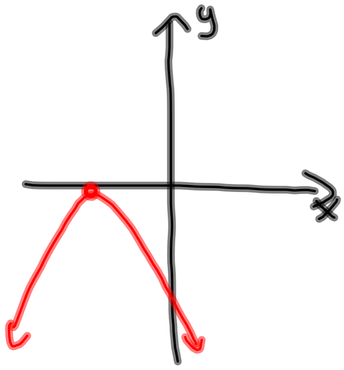
b) $y = -3x^2 - 2x - 1$

no x-intercepts



c) $y = x^2 + 16x + 64$

1 x-intercept



Example 3

Find the value of *c* in the quadratic equation $x^2 + 4x + c = 0$, such that the equation has:

a) two real roots $D > 0$
b) one real root $D = 0$
c) no real roots (two imaginary roots) $D < 0$

Solution

a) For two real roots, the discriminant must be greater than 0.

$$b^2 - 4ac > 0$$
$$(4)^2 - 4(1)(c) > 0$$
$$16 - 4c > 0$$
$$-4c > -16$$
$$\mathbf{c < 4}$$

* **Remember, dividing by a negative on both sides switches the inequality!**
All values of *c* less than 4 results in two real solutions.

b) For one real root, the discriminant must equal 0.

$$b^2 - 4ac = 0$$
$$(4)^2 - 4(1)(c) = 0$$
$$16 - 4c = 0$$
$$-4c = -16$$
$$\mathbf{c = 4}$$

The only value of *c* that will result in the equation having one real root is 4.

c) For no real root, the discriminant must be less than 0.

$$b^2 - 4ac < 0$$
$$(4)^2 - 4(1)(c) < 0$$
$$16 - 4c < 0$$
$$-4c < -16$$
$$\mathbf{c > 4}$$

Switch Inequality!
All values of *c* greater than 4 will result in the equation having two imaginary roots.

Assignment

$D = b^2 - 4ac$

For each of the following quadratic equations, evaluate the discriminant, then predict what type of roots the equation will have. *Do not solve the equation.*

35. $2x^2 + 3x - 10 = 0$

36. $9x^2 - 12x + 4 = 0$

37. $3x^2 - 7x + 5 = 0$

35 $a = 2$
$b = 3$
$c = -10$

$D = (3)^2 - 4(2)(-10)$
$= 9 + 80$
$= 89$

2 Real Roots

For each of the following quadratic functions, evaluate the discriminant, then predict how many x-intercepts the parabola will have.

38. $y = x^2 + 8x + 16$

39. $y = 5x^2 + 2x + 1$

40. $y = 7x^2 + 6x$

41. $y = x^2 - 13x + 42$

42. For what value of c will the equation $\mathbf{3x^2 + 4x + c = 0}$ have:

a) two real roots

b) one real root

c) two imaginary roots