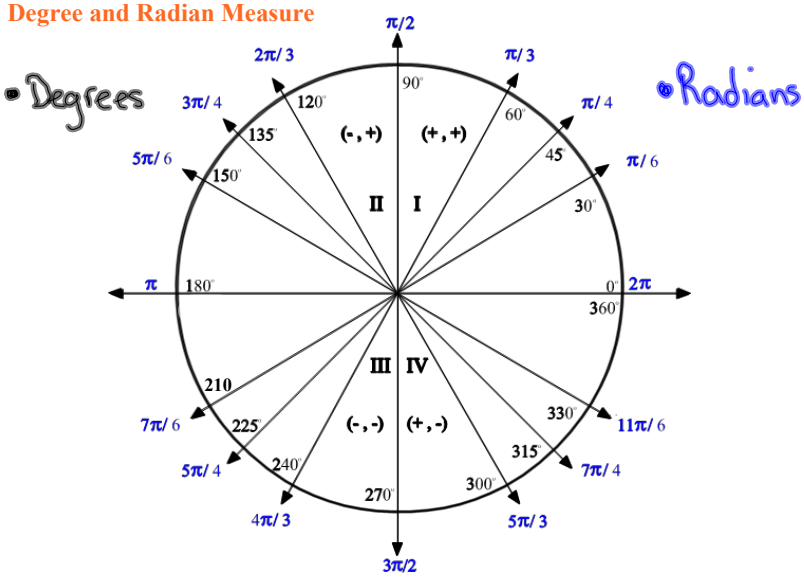
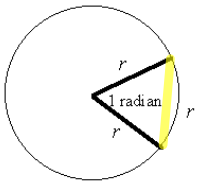
Degree and Radian Measure



A radian is the angle subtended by an arc of length r (radius)



$$\theta = \frac{a}{r}$$ ← arc length; ← radius

$$360° = \frac{2\pi r}{r}$$

Degrees Radians

$360^{\circ} = 2\pi$

$180° = \pi$

$1° = \frac{\pi}{180}$ radians	$1 \text{ rad} = \frac{180}{\pi}$

Degrees to Radians:

Ex: $30° \times \frac{\pi}{180°}$

$\frac{30\pi}{180}$

$\frac{\pi}{6}$

Radians to Degrees:

Ex. $\frac{3\pi}{2} \times \frac{180}{\pi}$

$\frac{540\pi}{2\pi}$

$270°$

Polar Coordinates

(Alternative form of graphing)

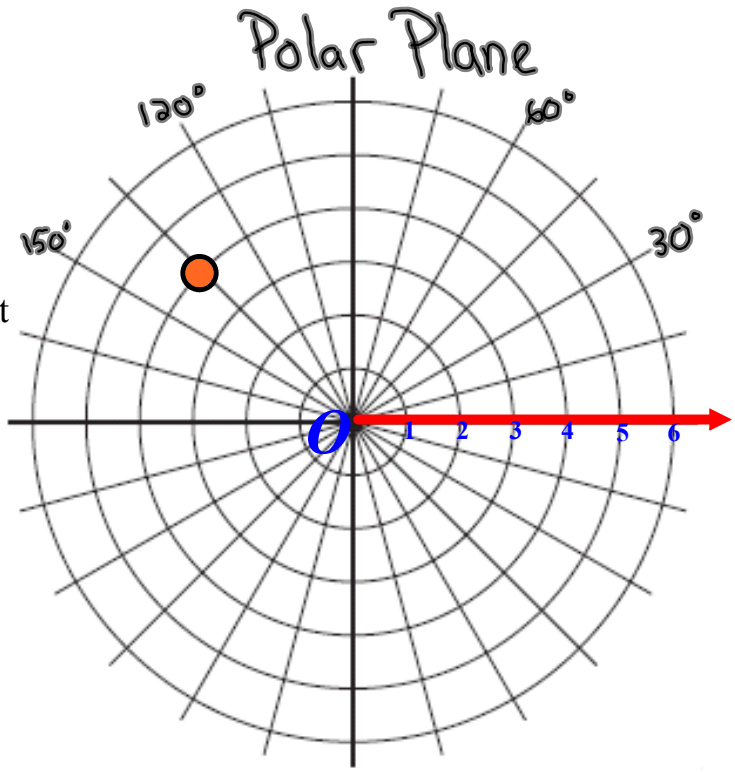
Consider a series of concentric circles having a common center, O, called the ***pole***. The ***polar axis*** is the horizontal ray drawn from the pole in a positive direction (to the right).

Any point "P" plotted on the graph is described by a directed distance ***r*** and by the angle that OP makes with the polar axis (we use θ to represent the angle).

(r, θ)

Let's look at the point ***P(4, 135°)***

or $P(4, \frac{3\pi}{4})$



Are there any other ways to represent the position of point ***P ?***

$(-4, -45°)$

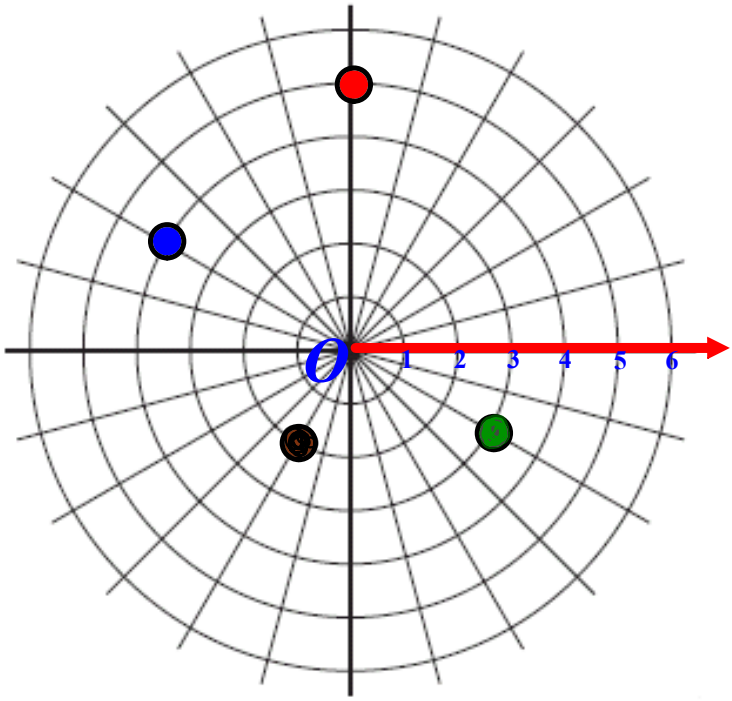
(4, -225°)

(4, 495°)

(-4, 315°)

Plot the following points:

- *(2, 240°)*
- *(4, -210°)*
- *(5, 450°)*
- *(-3, 150°)*



Converting ***Rectangular Coordinates*** to ***Polar Coordinates***

(x, y) ⟶ ***(r, θ)***

or (a, b)

To do so we must relate the polar coordinate system to the cartesian system by letting the polar axis coincide with the x axis so that the pole is at the origin.

1. Find the radius ***r,*** using the Pythagorean relationship $r = \sqrt{x^2 + y^2}$

2. Find the related angle, α, using $\alpha = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

   ref. angle

3. Find the angle, θ, by determining the quadrant in which the terminal arm is located and using the related angle.

☆

180-α	α
180+α	360-α

4. The polar coordinates are ***(r, θ)***

Let's try an example

Convert ***P(-4, 2)*** to Polar form

$(x,y) \longrightarrow (r,\theta)$

$x = -4$
$y = 2$ } Quad 2

① Find r:

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(-4)^2+(2)^2}$

$r = \sqrt{16+4}$

$r = \sqrt{20}$

$r = 2\sqrt{5}$

② Find α:

$\alpha = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

$\alpha = \tan^{-1}\left(\frac{2}{4}\right)$

$\alpha = 26.57°$

③ Find θ (Quad 2)

$\theta = 180 - 26.57$

$\theta = 153.43°$

④ Polar Form (r,θ)

$(2\sqrt{5}, 153.43°)$

Converting ***Polar Coordinates*** to ***Rectangular Coordinates***

$(r, \theta) \longrightarrow (x, y)$

Recall:

$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

Therefore:

$$\boxed{x = r\cos\theta}$$

$$\boxed{y = r\sin\theta}$$

1. Find ***x***
2. Find ***y***
3. The ***Rectangular Coordinates*** are ***(x, y)***

Example

$(r,\theta) \longrightarrow (x,y)$

Convert $C(4, 150^\circ)$ to Rectangular form

$r = 4$

$\theta = 150^\circ$

① $x = r\cos\theta$

$x = 4\cos 150^\circ$

$x = -3.46$

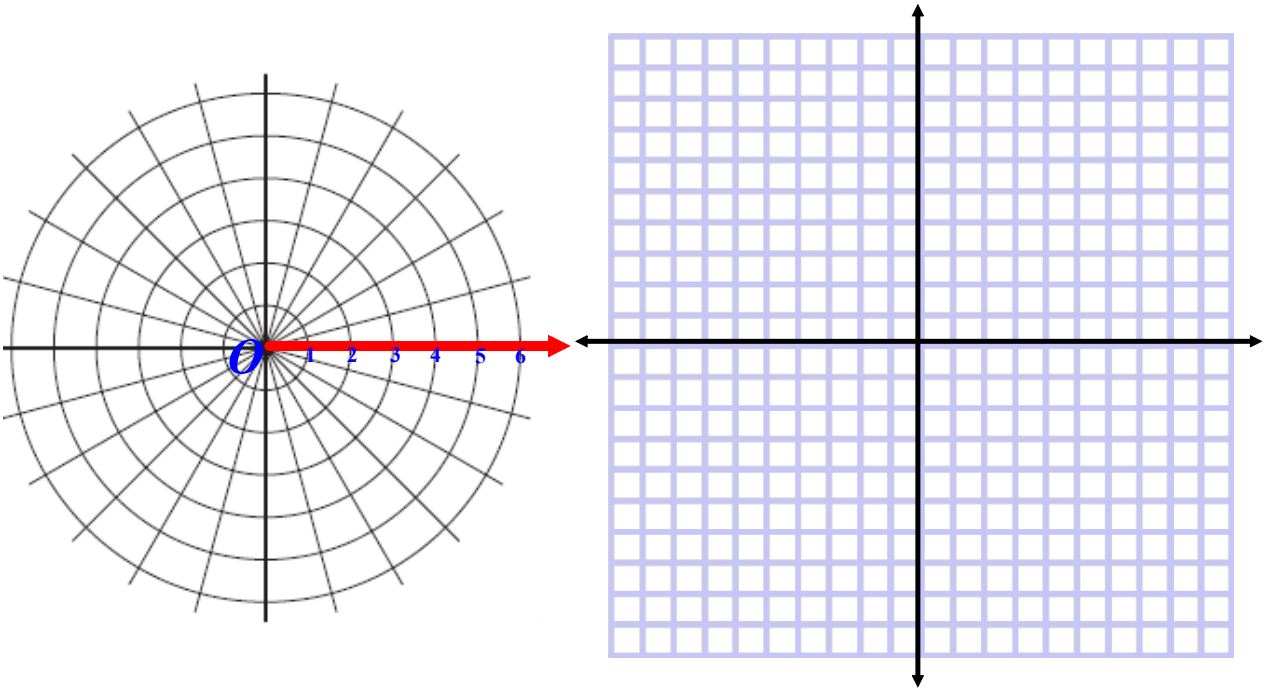
② $y = r\sin\theta$

$y = 4\sin 150^\circ$

$y = 2$

③ $(-3.46, 2)$

Homework


O
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