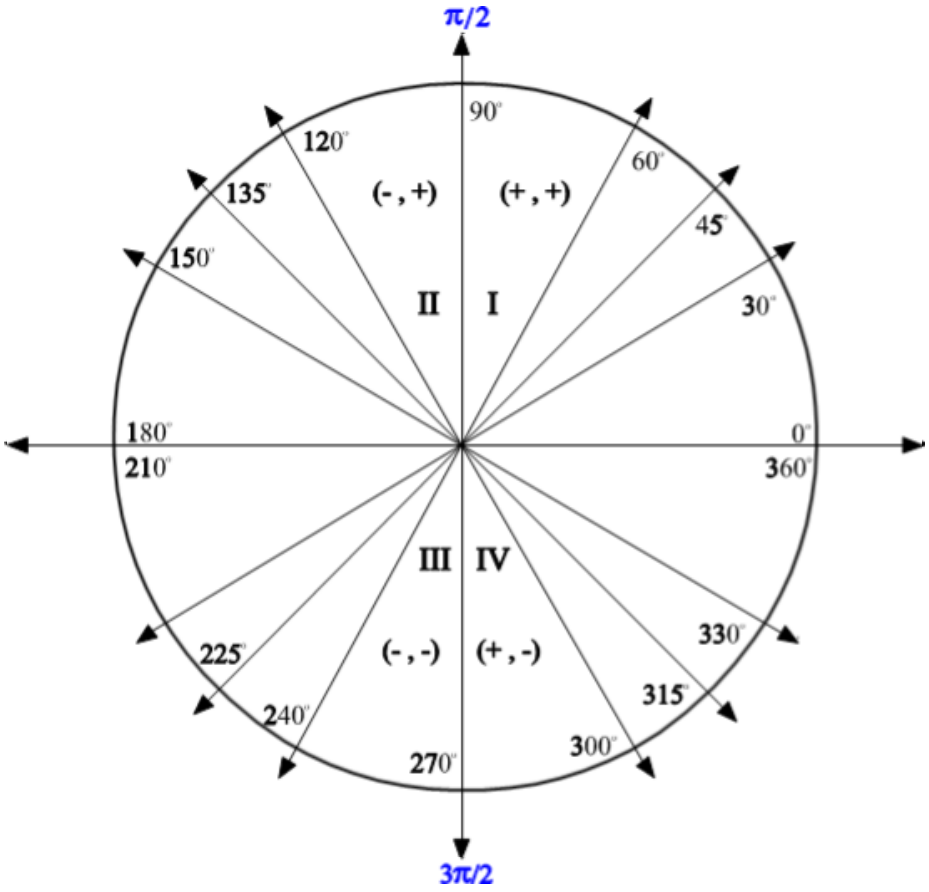
Degree and Radian Measure



Rectangular

Convert the point *(5, -12)* to Polar coordinates

$(x, y) \longrightarrow (r, \theta)$

1. Find the radius ***r,*** using the Pythagorean relationship $r = \sqrt{x^2 + y^2}$
2. Find the related angle, α, using $\alpha = \tan^{-1}\left(\frac{|y|}{|x|}\right)$
3. Find the angle, θ, by determining the quadrant in which the terminal arm is located and using the related angle.

★

180-α	α
180+α	360-α

Remember from last semester

4. The polar coordinates are ***(r, θ)***

$x = 5$
$y = -12$
} Quad 4

① $r = \sqrt{x^2 + y^2}$

$r = \sqrt{(5)^2 + (-12)^2}$

$r = \sqrt{25 + 144}$

$r = \sqrt{169}$

$r = 13$

② $\alpha = \tan^{-1}\left(\frac{|y|}{|x|}\right)$

$\alpha = \tan^{-1}\left(\frac{12}{5}\right)$

$\alpha = 67.4°$

③ Quad 4

$\theta = 360 - \alpha$

$\theta = 360° - 67.4°$

$\theta = 292.6°$

④ $(13, 292.6°)$

$(r, \theta) \longrightarrow (x, y)$

Convert the point $\left(4, \frac{4\pi}{3}\right)$ to rectangular coordinates.

or $(4, 240°)$ {Convert to Degrees $\rightarrow \left(\frac{180}{\pi}\right)$}

$\frac{4\pi}{3} \times \frac{180}{\pi} = 240°$

$x = r\cos\theta$

$y = r\sin\theta$

① $x = 4\cos 240°$

$x = -2$

② $y = 4\sin 240°$

$y = -3.46$ or $-2\sqrt{3}$

③ $(-2, -3.46)$

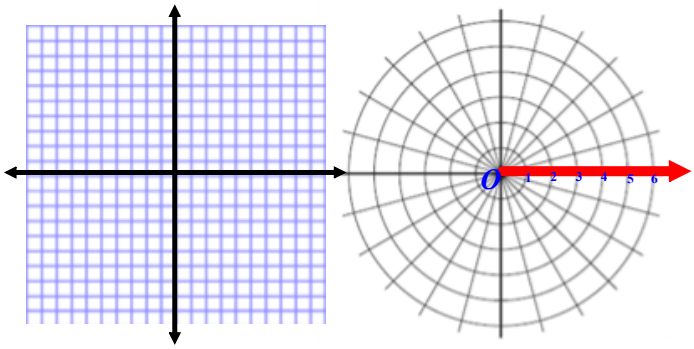
Converting Complex numbers in rectangular form to ***Polar Form***

$(a+bi) \longrightarrow r\,\text{cis}\,\theta$

Convert to Polar form

$5-12i$

$a=5$
$b=-12$ } Quad 4



① $r=\sqrt{a^2+b^2}$

$r=\sqrt{(5)^2+(12)^2}$

$r=\sqrt{25+144}$

$r=\sqrt{169}$

$r=13$

② $\alpha=\tan^{-1}\left(\frac{|b|}{|a|}\right)$

$\alpha=\tan^{-1}\left(\frac{12}{5}\right)$

$\alpha=67.4°$

③ Quad 4

$\theta=360-\alpha$

$\theta=360°-67.4°$

$\theta=292.6°=293°$

④ $13\,\text{cis}\,293°$

↳ $13(\cos 293°+i\sin 293°)$

↳ $13\cos 293°+13i\sin 293°$

A complex number in the form, $r\cos\theta + ri\sin\theta$, can be factored as $r(\cos\theta + i\sin\theta)$
The factored expression is usually shortened to $rcis\theta$

Polar

$r = 5 \quad \theta = 90°$

$5cis90°$ → shortened Form (Most common)

$= 5(\cos 90° + i\sin 90°)$ → Factored Form

$= 5\cos 90° + 5i\sin 90°$ → Expanded Form

$= 5(0) + 5i(1)$

$= 0 + 5i$

* Quadrantal Angle you can simplify

↑ rectangular form

Example

Covert from Polar to Retangular form

$(r\text{cis}\theta) \longrightarrow (a+bi)$

$6\text{cis}150^\circ$

$a = r\cos\theta$

$b = r\sin\theta$

$r = 6$

$\theta = 150$

① $a = r\cos\theta$

$a = 6\cos 150^\circ$

$a = -5.2$

or $-3\sqrt{3}$

② $b = r\sin\theta$

$b = 6\sin 150^\circ$

$b = 3$

③ $-5.2 + 3i$

Homework

Do # 19, 21

$(a+bi) \longrightarrow r\,\text{cis}\,\theta$

⑲ b) $-5\sqrt{3}+5i$ $\quad a=-5\sqrt{3}$, $b=5$ } Quad 2

① $r=\sqrt{a^2+b^2}$

$r=\sqrt{(-5\sqrt{3})^2+(5)^2}$

$r=\sqrt{75+25}$

$r=\sqrt{100}$

$r=\underline{10}$

② $\alpha=\tan^{-1}\left(\frac{|b|}{|a|}\right)$

$\alpha=\tan^{-1}\left(\frac{5}{5\sqrt{3}}\right)$

$\alpha=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\alpha=30°$

Quad 2

③ $\theta=180-\alpha$

$\theta=180°-30°$

$\theta=\underline{150°}$

④ $10\,\text{cis}\,150°$