Polar Coordinates

1. Convert 4 - 3i to Polar form

Find the radius *r*, using the Pythagorean relationship $r = \sqrt{a^2 + b^2}$

Find the related angle, α, using $\alpha = \tan^{-1}\left(\frac{|b|}{|a|}\right)$

Find the angle, θ, by determining the quadrant in which the terminal arm is located and using the related angle.

★

180-α	α
180+α	360-α

Remember from last semester

The polar form is ***rcis***θ

2. Convert 2cis47° to rectangular form

$a = r\cos\theta$

$b = r\sin\theta$

① $\boxed{4-3i}$

$a=4$, $b=-3$ } Quad 4

$r=\sqrt{a^2+b^2}$
$r=\sqrt{(4)^2+(-3)^2}$
$r=\sqrt{16+9}$
$r=\sqrt{25}$
$r=5$

$\alpha=\tan^{-1}\left(\frac{|b|}{|a|}\right)$
$\alpha=\tan^{-1}\left(\frac{3}{4}\right)$
$\alpha=36.9°$

Quad 4
$\theta=360-\alpha$
$\theta=360-36.9°$
$\theta=323.1°$

$\boxed{5cis323.1°}$

② $\boxed{2cis47°}$

$r=2$
$\theta=47°$

$a=r\cos\theta$
$a=2\cos47°$
$a=1.36$

$b=r\sin\theta$
$b=2\sin47°$
$b=1.46$

$\boxed{1.36+1.46i}$

Operations with Complex Numbers in Polar Form (rcisθ)

You may have noticed a shortcut when multiplying complex numbers in Polar form.

- When Multiplying, *multiply* the "r" values and *add* the arguments.
- When Dividing you *divide* the "r" values and *subtract* the arguments

Argument:

The angle from the positive real axis to the position vector representing a complex number in the complex plane. If the number is written in ***polar form*** as ***rcis***θ then θ is the argument and "r" is the modulus.

Examples

$2\times3=6$ $\quad$ $150+200=350°$

$(2cis150°)(3cis200°) = \boxed{6cis350°}$

$(2\sqrt{2}cis60°)(3\sqrt{8}cis240°) = \boxed{24cis300°}$

$(3cis150°)(5cis240°) = 15cis390° = \boxed{15cis30°}$

$\dfrac{45cis120°}{3cis190°} = 15cis(-70°) = \boxed{15cis290°}$

De Moivre's Theorem

(must be in polar form)

$$(r\,cis\,\theta)^n = r^n\,cis\,n\theta$$

Ex: Rectangular (a+bi)

$$(1+i\sqrt{3})^9 = (2cis60°)^9 = 2^9 cis(9\times60°) = 512cis540°$$

polar $= \boxed{512cis180°}$

rectangular $= \boxed{-512+0i}$

$a=1$, $b=\sqrt{3}$ } Quad 1

$r=\sqrt{(1)^2+(\sqrt{3})^2}$

$\alpha=\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\alpha=60°$

Quad 1

$\theta=\alpha$

$\theta=60°$

$2cis60°$

Homework

$$\frac{\pi}{4} \times \frac{180}{\pi} = \frac{180\pi}{4\pi}$$

$$= 45°$$

② a) $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^6$

$$\left(1\cos\frac{\pi}{4} + 1i\sin\frac{\pi}{4}\right)^6$$

$$\left(1\text{cis}\frac{\pi}{4}\right)^6$$

or $\left(1\text{cis}45°\right)^6$