Convert the following complex numbers from ***rectangular*** to **polar form**

$(5\sqrt{3} - 5i)$

$a = 5\sqrt{3}$, $b = -5$ } Quad 4

① $r = \sqrt{(5\sqrt{3})^2 + (-5)^2}$

$r = \sqrt{75 + 25}$

$r = 10$

② $\alpha = \tan^{-1}\left(\frac{5}{5\sqrt{3}}\right)$

$\alpha = 30°$

③ Quad 4

$\theta = 360 - \alpha$

$\theta = 360° - 30°$

$\theta = 330°$

④ $10\text{cis}\,330°$

$(1 - i\sqrt{3})$

$a = 1$, $b = -\sqrt{3}$ } Quad 4

① $r = \sqrt{(1)^2 + (\sqrt{3})^2}$

$r = \sqrt{1 + 3}$

$r = 2$

② $\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\alpha = 60°$

③ Quad 4

$\theta = 360 - \alpha$

$\theta = 360 - 60$

$\theta = 300°$

④ $2\text{cis}\,300°$

Multiply your answers

$(10\text{cis}\,330°)(2\text{cis}\,300°)$

$20\text{cis}\,630°$

$20\text{cis}\,270°$ Polar

$0 - 20i$ Rectangular

Divide your answers

$\frac{10\text{cis}\,330°}{2\text{cis}\,300°}$

$5\text{cis}\,30°$ Polar

$4.33 + 2.5i$ Rec.

1c) $\dfrac{(4-4i\sqrt{3})(2\sqrt{3}+2i)(1+i)}{(5-5i)(-\sqrt{3}+i)}$

$a=4$, $b=-4\sqrt{3}$; $r=\sqrt{16+48}$, $r=\sqrt{64}$, $r=8$; $\alpha=\tan^{-1}\left(\frac{4\sqrt{3}}{4}\right)$, $\alpha=60^\circ$; Quad 4, $\theta=300^\circ$

$a=2\sqrt{3}$, $b=2$; $r=\sqrt{12+4}$, $r=4$; $\alpha=\tan^{-1}\left(\frac{2}{2\sqrt{3}}\right)$, $\alpha=30^\circ$; Quad 1, $\theta=30^\circ$

$a=1$, $b=1$; $r=\sqrt{1+1}$, $r=\sqrt{2}$; $\alpha=\tan^{-1}\left(\frac{1}{1}\right)$, $\alpha=45^\circ$; Quad 1, $\theta=45^\circ$

$a=5$, $b=-5$; $r=\sqrt{25+25}$, $r=\sqrt{50}$, $r=5\sqrt{2}$; $\alpha=\tan^{-1}\left(\frac{5}{5}\right)$, $\alpha=45^\circ$; Quad 4, $\theta=315^\circ$

$a=-\sqrt{3}$, $b=1$; $r=\sqrt{3+1}$, $r=2$; $\alpha=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $\alpha=30^\circ$; Quad 2, $\theta=150^\circ$

$$\frac{(8\,\mathrm{cis}\,300^\circ)(4\,\mathrm{cis}\,30^\circ)(\sqrt{2}\,\mathrm{cis}\,45^\circ)}{(5\sqrt{2}\,\mathrm{cis}\,315^\circ)(2\,\mathrm{cis}\,150^\circ)}$$

$$\frac{32\sqrt{2}\,\mathrm{cis}\,375^\circ}{10\sqrt{2}\,\mathrm{cis}\,465^\circ}$$

$$3.2\,\mathrm{cis}(-90^\circ)$$

$\boxed{3.2\,\mathrm{cis}\,270^\circ}$ Polar

$3.2\cos 270^\circ + 3.2i\sin 270^\circ$

$3.2(0) + 3.2i(-1)$

$\boxed{-3.2i}$ Rectangular

You may have noticed a shortcut when multiplying and dividing complex numbers in Polar form.

For Multiplication
Multiply the "r" values and add the arguments.

For Division
Divide the "r" values and subtract the arguments.

De Moivre's Theorem

$$(rcis\theta)^n = r^n cisn\theta$$

Evaluate

$$(2\cos\frac{\pi}{6} + 2i\sin\frac{\pi}{6})^5$$

$(2cis\frac{\pi}{6})^5$

$2^5 cis(5\cdot\frac{\pi}{6})$

$32cis\frac{5\pi}{6}$

In Degrees:

$\frac{\pi}{6}\cdot\frac{180}{\pi} = 30°$

$(2cis30°)^5$

$2^5 cis(5\cdot30)$

$32cis150°$

Remember $(rcis\theta)^n = r^n cisn\theta$

$(3-4i)^5$

$a = 3$
$b = -4$ } Quad 4

$r = \sqrt{(3)^2 + (-4)^2}$
$r = \sqrt{9+16}$
$r = 5$

$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$
$\alpha = 53°$

Quad 4
$\theta = 360 - \alpha$
$\theta = 360 - 53°$
$\theta = 307°$

$5cis307°$

$(5cis307°)^5$

$5^5 cis(5 \cdot 307°)$

$3125 cis 1535°$

$3125 cis 95°$ Polar

$-372.4 + 3113.1i$ Rectangular

Homework

⑧ Ex $\dfrac{(4-3i)^5(1-i)^3}{(5\sqrt{3}+5i)^3}$ $\quad (r\,\text{cis}\,\theta)^n = r^n\,\text{cis}\,n\theta$

$a=4 \quad b=-3 \quad (\text{Quad } 4)$

$r=\sqrt{16+9}$ $\quad \alpha=\tan^{-1}\left(\frac{3}{4}\right)$ $\quad \theta=360-36.9$ $\quad \boxed{5\,\text{cis}\,323.1^\circ}$

$r=\sqrt{25}$ $\quad \alpha=36.9^\circ$ $\quad \theta=323.1^\circ$

$r=5$

$a=1 \quad b=-1 \quad (\text{Quad } 4)$

$r=\sqrt{1+1}$ $\quad \alpha=\tan^{-1}\left(\frac{1}{1}\right)$ $\quad \theta=360-45$ $\quad \boxed{\sqrt{2}\,\text{cis}\,315^\circ}$

$r=\sqrt{2}$ $\quad \alpha=45^\circ$ $\quad \theta=315^\circ$

$a=5\sqrt{3} \quad b=5 \quad (\text{Quad } 1)$

$r=\sqrt{(5\sqrt{3})^2+(5)^2}$ $\quad \alpha=\tan^{-1}\left(\frac{5}{5\sqrt{3}}\right)$ $\quad \theta=30^\circ$ $\quad \boxed{10\,\text{cis}\,30^\circ}$

$r=\sqrt{75+25}$ $\quad \alpha=30^\circ$

$r=10$

$$\frac{(5\,\text{cis}\,323.1)^5(\sqrt{2}\,\text{cis}\,315)^3}{(10\,\text{cis}\,30)^3}$$

$$\frac{[5^5\,\text{cis}(5\cdot 323.1)][\sqrt{2}^3\,\text{cis}(3\cdot 315)]}{10^3\,\text{cis}(3\cdot 30)}$$

$$\frac{(3125\,\text{cis}\,1615.5)(2\sqrt{2}\,\text{cis}\,945)}{100\,\text{cis}\,60^\circ}$$

$$\frac{6250\sqrt{2}\,\text{cis}\,2560.5^\circ}{100\,\text{cis}\,60^\circ}$$

$$62.5\sqrt{2}\,\text{cis}\,2500.5^\circ$$

$$\boxed{62.5\sqrt{2}\,\text{cis}\,340.5^\circ}$$