SOLUTIONS ⇒ Chapter 5 - Chapter Test

MULTIPLE CHOICE

1. Suppose you graph the linear inequality $2x + y < 4$. Which set of statements describes the graph of the linear inequality?

   A. The boundary line is a solid line. The plane is shaded above the line.

   B. The boundary line is a dashed line. The plane is shaded above the line.

   C. The boundary line is a dashed line. The plane is shaded below the line.

   D. The boundary line is a solid line. The plane is shaded below the line.

Equation of boundary: $2x+y=4$

2 points located on the boundary:

x-int:

$2x+0=4$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{4}{2}$

$x=2$

y-int:

$2(0)+y=4$

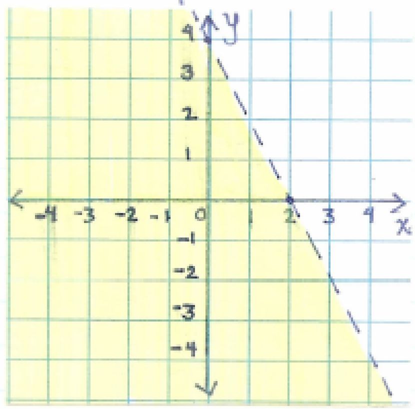
$y=4$

Test Point: (0,0)

L.S.	R.S.
$2x+y$	4
$2(0)+0$	
0	

0<4, therefore (0,0) is located in solution region.

Graph:



Test Point; (0,0)

L.S	R.S.
$y-x$	-2
$0-0$	
0	

$0 > -2$, (0,0) is located in solution region

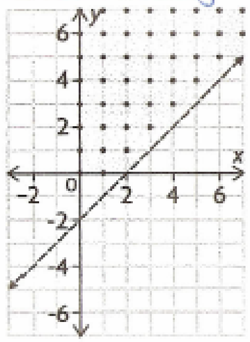
2. Which linear inequality is shown in the graph?

A. $\{(x, y) \mid y - x \geq -2, x \in W, y \in W\}$

B. $\{(x, y) \mid y - x > -2, x \in W, y \in W\}$

C. $\{(x, y) \mid y - x \geq -2, x \in R, y \in R\}$

D. $\{(x, y) \mid y - x > -2, x \in I, y \in I\}$



3. Which is a solution to the system of linear inequalities?

$\{(x, y) \mid 2x + y > 5, x \in I, y \in I\}$

$\{(x, y) \mid y - x < 4, x \in I, y \in I\}$

A. (3, 1) **B.** (4.5, 0) **C.** (−2, 1) **D.** (−3, −1)

For (3,1):

L.S.	R.S.
$2x+y$	5
$2(3)+1$	
$6+1$	
7	

✓

L.S	R.S
$y-x$	4
$3-1$	
2	

✓

For (4.5, 0):

L.S.	R.S
$2x+y$	5
$2(4.5)+0$	
$9+0$	
9	

✗

L.S.	R.S.
$y-x$	4
$0-4.5$	
-4.5	

✓

For (−2,1):

L.S.	R.S
$2x+y$	5
$2(-2)+1$	
$-4+1$	
-3	

✗

L.S	R.S
$y-x$	4
$1-(-2)$	
3	

✓

For (−3,−1):

L.S.	R.S
$2x+y$	5
$2(-3)-1$	
$-6-1$	
-7	

✗

L.S	R.S.
$y-x$	4
$-1-(-3)$	
2	

✓

4. Consider this system:

$\{(x, y) \mid 3y + x \geq 3, x \in R, y \in R\}$

$\{(x, y) \mid x - y < 4, x \in R, y \in R\}$

The boundaries for the two inequalities intersect at the point $(3.75, -0.25)$. Which statement about this point is most accurate?

A. The point is not in the solution set, because its coordinates are not whole numbers.

B. The point is in the solution set, because it lies on both boundaries.

C. The point is not in the solution set, because one of the inequality signs is $<$ or $>$.

D. The point is in the solution set, because one of the inequality signs is $\leq$ or $\geq$.

5. A sports equipment manufacturer produces snowboards and skis. It takes 4 h to cut and mould each board and 1 h to put on the finishes. It takes 4 h to cut and mould and 2 h to put on the finishes for a pair of skis. The total number of snowboards and pairs of skis produced per day is at most 15.

Let a represent the number of snowboards and b represent the number of pairs of skis made in one day or less. What are the restrictions on a and b?

A. no restrictions **B.** $a \in N, b \in N$ **C.** $a \in I, b \in I$ **D.** $a \in W, b \in W$

6. Which algebraic model represents the situation in question 5?

A. $\{(a, b) \mid a \geq 0, b \geq 0, a + b \leq 15, a \in R, b \in R\}$

$\{(a, b) \mid a \geq 0, b \geq 0, 5a + 6b \leq 24, a \in R, b \in R\}$

B. $\{(a, b) \mid a \geq 0, b \geq 0, a + b \leq 15, a \in I, b \in I\}$

$\{(a, b) \mid a \geq 0, b \geq 0, 5a + 6b \leq 24, a \in I, b \in I\}$

C. $\{(a, b) \mid a \geq 0, b \geq 0, a + b \leq 15, a \in W, b \in W\}$

$\{(a, b) \mid a \geq 0, b \geq 0, 5a + 6b \leq 24, a \in W, b \in W\}$

D. $\{(a, b) \mid a \geq 0, b \geq 0, a + b \leq 4, a \in N, b \in N\}$

$\{(a, b) \mid a \geq 0, b \geq 0, 5a + 6b \leq 24, a \in N, b \in N\}$

8. Consider this system of linear inequalities:

$y + 3x \geq 9$

$y < 2x - 3$

a) Determine the point of intersection for the system of linear inequalities.

Point of intersection: (2.4, 1.8)

b) Will the point be a solid dot or an open dot on a graph of the system?

A(n) open dot

Equations of the boundaries:

→ $y+3x=9$ → $y=2x-3$

2 points located on each boundary (x-int & y-int):

→ $y+3x=9$

x-int:
$0+3x=9$
$\frac{\cancel{3}x}{\cancel{3}}=\frac{9}{3}$
$x=3$

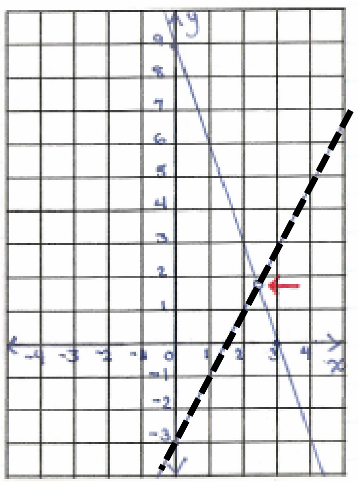
y-int:
$y+3(0)=9$
$y=9$

→ $y=2x-3$

x-int:
$0=2x-3$
$\frac{3}{2}=\frac{\cancel{2}x}{\cancel{2}}$
$\frac{3}{2}=x$
$1.5=x$

y-int:
$y=2(0)-3$
$y=0-3$
$y=-3$

GRAPH (to determine point of intersection):



* Shaded region is not shown.

Point of Intersection located at approx. (2.4, 1.8)

9. Graph each system. Determine a solution for each.

a) $\{(x, y) \mid y \geq 0.5x, x \in R, y \in R\}$

$\{(x, y) \mid x + y < 7, x \in R, y \in R\}$

Equations of the boundaries:

→ $y = 0.5x$ → $x + y = 7$

2 points on each boundary:

→ $y = 0.5x$ (x-int & y-int will both be 0) → $x + y = 7$

If $x = 2$:

$y = 0.5(2)$ (2, 1)

$y = 1$

x-int: $x + 0 = 7$, $x = 7$

y-int: $0 + y = 7$, $y = 7$

Test Points:

→ $y \geq 0.5x$; (6, 0)

L.S	R.S.
y	$0.5x$
0	$0.5(6)$
0	3

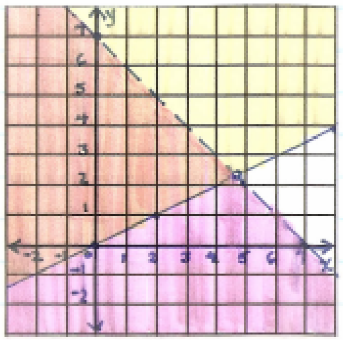
Since 0 is not ≥ 3, (0,0) is not located in solution region.

→ $x + y < 7$; (0, 0)

L.S	R.S
$x+y$	7
$0+0$	
0	

Since 0 < 7, (0,0) is located in the solution region.

GRAPH:



Solution:

For example: (2, 4)

b) $\{(x, y) \mid y - 2x > 2, x \in W, y \in W\}$
$\{(x, y) \mid x + 2y < 12, x \in W, y \in W\}$

Equations of the boundaries:

→ $y - 2x = 2$ → $x + 2y = 12$

2 points on each boundary:

→ $y - 2x = 2$

x-int:
$0 - 2x = 2$
$\frac{-\cancel{2}x}{-\cancel{2}} = \frac{2}{-2}$
$x = -1$

y-int:
$y - 2(0) = 2$
$y - 0 = 2$
$y = 2$

→ $x + 2y = 12$

x-int:
$x + 2(0) = 12$
$x + 0 = 12$
$x = 12$

y-int:
$0 + 2y = 12$
$\frac{\cancel{2}y}{\cancel{2}} = \frac{12}{2}$
$y = 6$

Test Points:

→ $y - 2x > 2$; $(0,0)$

L.S	R.S
$y - 2x$	2
$0 - 2(0)$	
$0 - 0$	
0	

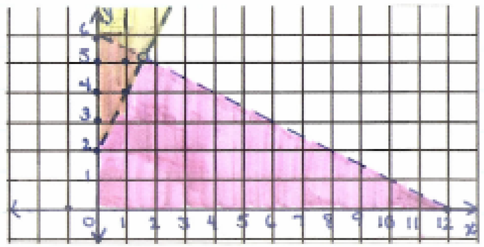
Since 0 is not > 2, (0,0) is not located in the solution region

→ $x + 2y < 12$

L.S.	R.S.
$x + 2y$	12
$0 + 2(0)$	
$0 + 0$	
0	

Since $0 < 12$, (0,0) is located in the solution region.

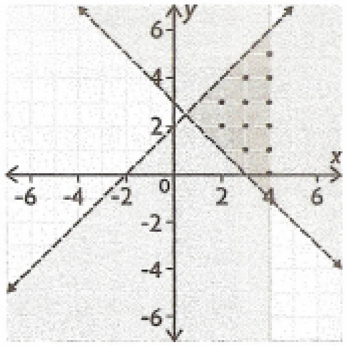
GRAPH:



Solution:

For example: (1,5)

10. The graph of a system of linear inequalities is shown, where the objective function is $P = 1.5x + 4y$.



a) Determine the vertices of the feasible region.

(0.5, 2.5), (4, -1), (4, 6)

b) What is the minimum solution for the system? (4, -1)

c) If P represents the amount of profit, in thousands of dollars, what is the minimum profit that can be made? $2,000

d) What is the maximum solution for the system? (4, 6)

e) If P represents the amount of profit, in thousands of dollars, what is the maximum profit that can be made? $30 000

* For (0.5, 2.5): $P = 1.5(0.5) + 4(2.5)$
$P = 0.75 + 10$
$P = 10.75$

For (4, -1): $P = 1.5(4) + 4(-1)$
$P = 6 - 4$
$P = 2$

For (4, 6): $P = 1.5(4) + 4(6)$
$P = 6 + 24$
$P = 30$

13. Jenna and Rhiana sell tacos and burritos from a food cart.

- No more than 50 tacos and 75 burritos can be made each day.
- Jenna and Rhiana can make no more than 125 items, in total, each day.
- It costs \$0.75 to make a taco and \$1.25 to make a burrito.

Create an optimization model and use it to determine the maximum and minimum costs to produce the food items.

Let t represent the number of tacos that can be made in a day.
Let b represent the number of burritos that can be made in a day.
Let C represent the cost of making the goods.

Restrictions: $t \in W$, $b \in W$

Constraints: $t \geq 0$, $b \geq 0$, $t \leq 50$, $b \leq 75$, $t + b \leq 125$.

Objective Function: $C = 0.75t + 1.25b$

Equations of the boundaries:

→ $t = 50$ → $b = 75$ → $t + b = 125$

2 points on each boundary (x-int & y-int):

→ $t = 50$ (O.K) → $b = 75$ (O.K.) → $t + b = 125$

* Vertical Line * Horizontal Line

t-int: $t + 0 = 125$, $t = 125$

b-int: $0 + b = 125$, $b = 125$

Test Points:

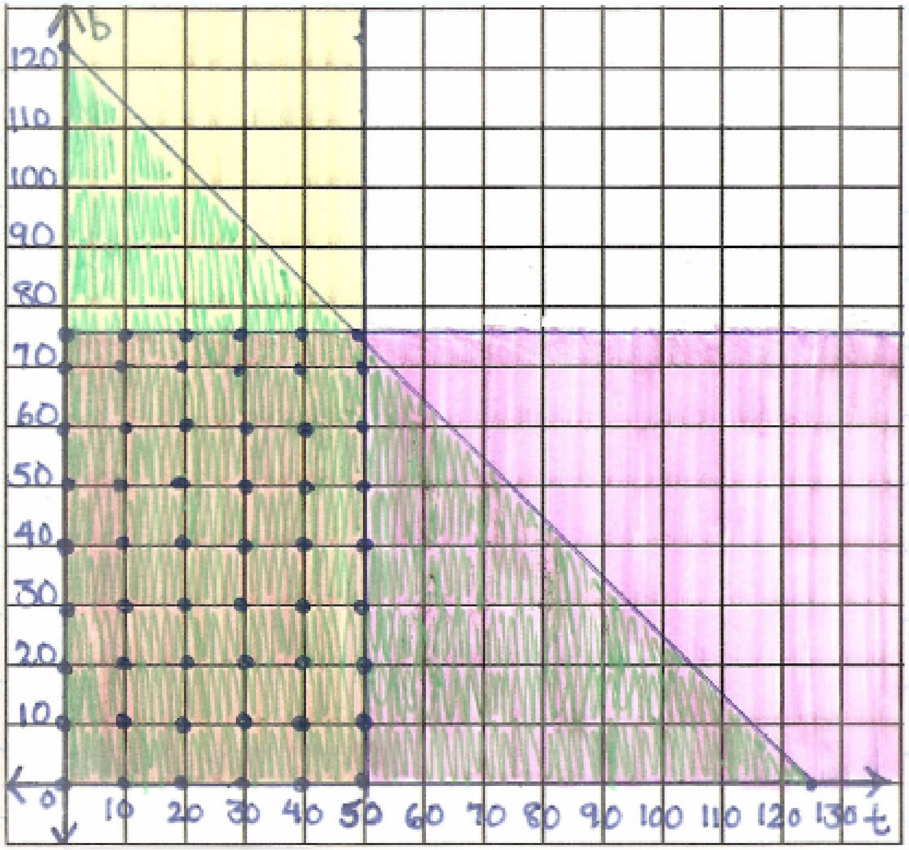
→ $t \le 50$ (O.K.) → $b \le 75$ → $t + b \le 125$; (0,0)

* Shaded to the left of the line * Shaded below the line.

L.S.	R.S
$t + b$	125
$0 + 0$	
0	

Since $0 \le 125$, (0,0) is located in the solution region.

GRAPH:



Vertices of feasible region:

(0,0), (0,75), (50,75) and (50,0)

For (0,0): $C = 0.75t + 1.25b$
$C = 0.75(0) + 1.25(0)$
$C = \$0$

For (0,75): $C = 0.75t + 1.25b$
$C = 0.75(0) + 1.25(75)$
$C = 0 + 93.75$
$C = \$93.75$

For (50,75): $C = 0.75t + 1.25b$
$C = 0.75(50) + 1.25(75)$
$C = 37.50 + 93.75$
$C = \$131.25$

For (50,0): $C = 0.75t + 1.25b$
$C = 0.75(50) + 1.25(0)$
$C = 37.50 + 0$
$C = \$37.50$.

* Minimum Cost => \$0 (0 tacos / 0 burritos)
Maximum Cost => \$131.25 (50 tacos / 75 burritos)