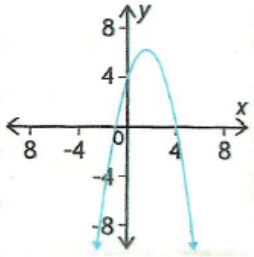
SOLUTIONS => 6.4 Factored Form of a Quadratic Function.

1. Match each quadratic function with its corresponding parabola.

i)

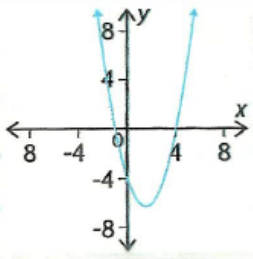


Match: f) $f(x) = (x+1)(4-x)$

$= (x+1)[-(-4+x)]$

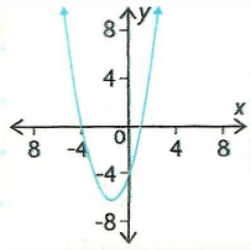
$f(x) = -(x+1)(x-4)$

ii)



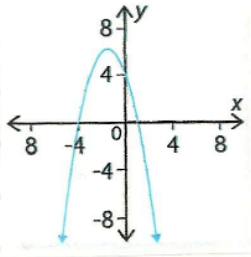
Match: b) $f(x) = (x+1)(x-4)$

iii)



Match: a) $f(x) = (x-1)(x+4)$

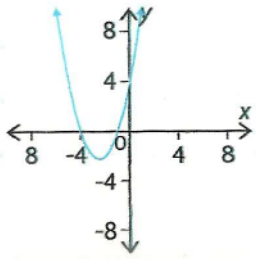
iv)



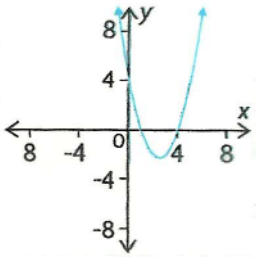
Match: e) $f(x) = (1-x)(x+4)$

$= -[(-1+x)](x+4)$

$= -(x-1)(x+4)$

v) 

Match: c) $f(x) = (x+1)(x+4)$

vi) 

Match: d) $f(x) = (x-1)(x-4)$

3. A quadratic function has an equation that can be written in the form $f(x) = a(x-r)(x-s)$. The graph of the function has x-intercepts $x=-2$ and $x=4$ and passes through point $(5,7)$. Write the equation of the quadratic function.

$f(x) = a(x-r)(x-s)$ $\quad r=-2,\ s=4,\ (5,7)$ (x, y)

$\Rightarrow y = a(x--2)(x-4)$

$\Rightarrow y = a(x+2)(x-4)$

$7 = a(5+2)(5-4)$

$7 = a(7)(1)$

$\frac{7}{7} = \frac{7a}{7}$

$1 = a$

To find "a" Substitute $x=5$ & $y=7$.

$\Rightarrow y = 1(x+2)(x-4)$

4. For each quadratic function, determine the x-intercepts, the y-intercept, the equation of the axis of symmetry, and the coordinates of the vertex of the graph.

{a-d}

a) $f(x) = (x-1)(x+1)$

x-intercepts: $x = r$ and $x = s$
$x = 1$ $x = -1$

y-intercept: $c = a \cdot r \cdot s$
$c = (1)(1)(-1)$
$c = -1$

Axis of Symmetry: $x = \frac{r+s}{2}$

$x = \frac{1 + -1}{2}$

$x = \frac{0}{2}$

$x = 0$

Vertex: $(x=0)$

$y = (x-1)(x+1)$ $(0, -1)$
$= (0-1)(0+1)$
$= (-1)(1)$
$= -1$

b) $f(x) = (x+2)(x+2)$

x-intercepts: $x = r$ and $x = s$
$x = -2$ $x = -2$ (SAME)

y-intercept: $c = a \cdot r \cdot s$
$c = (1)(-2)(-2)$
$c = 4$

Axis of Symmetry: $x = \frac{r+s}{2}$

$$x = \frac{-2+-2}{2}$$

$$x = \frac{-4}{2}$$

$$x = -2$$

Vertex: ($x = -2$)

$y = (x+2)(x+2)$ $(-2, 0)$
$y = (-2+2)(-2+2)$
$y = (0)(0)$
$y = 0$

c) $f(x) = (x-3)(x-3)$

x-intercepts: $x=r$ and $x=s$
$x=3$ $x=3$ (SAME)

y-intercept: $c = a \cdot r \cdot s$
$c = (1)(3)(3)$
$c = 9$

Axis of Symmetry: $x = \frac{r+s}{2}$

$x = \frac{3+3}{2}$

$x = \frac{6}{2}$

$x = 3$

Vertex: $(x=3)$ $y = (x-3)(x-3)$ (3,0)
$y = (3-3)(3-3)$
$y = (0)(0)$
$y = 0$

d) $f(x) = -2(x-2)(x+1)$

x-intercepts: $x = r$ and $x = s$
$x = 2$ $x = -1$

y-intercept: $c = a \cdot r \cdot s$
$c = (-2)(2)(-1)$
$c = 4$

Axis of Symmetry: $x = \frac{r+s}{2}$

$x = \frac{2+-1}{2}$

$x = \frac{1}{2}$

$x = 0.5$

Vertex: ($x = 0.5$)

$y = -2(x-2)(x+1)$ (0.5, 4.5)
$= -2(0.5-2)(0.5+1)$
$= -2(-1.5)(1.5)$
$= 4.5$

10. For each quadratic function below

{a}

i) use partial factoring to determine two points that are the same distance from the axis of symmetry.

ii) determine the coordinates of the vertex.

iii) Sketch the graph.

a) i) $f(x)=x^2+4x-6$

$f(x)=x(x+4)-6$

$x=0$ or $x+4=0$

$x=-4$

$f(0)=-6$ $f(-4)=-6$

Two Points: $(0,-6)$ & $(-4,-6)$

ii) To locate the vertex:

$x=\frac{0+-4}{2}$

$x=\frac{-4}{2}$

$x=-2$

$f(-2)=(-2)^2+4(-2)-6$

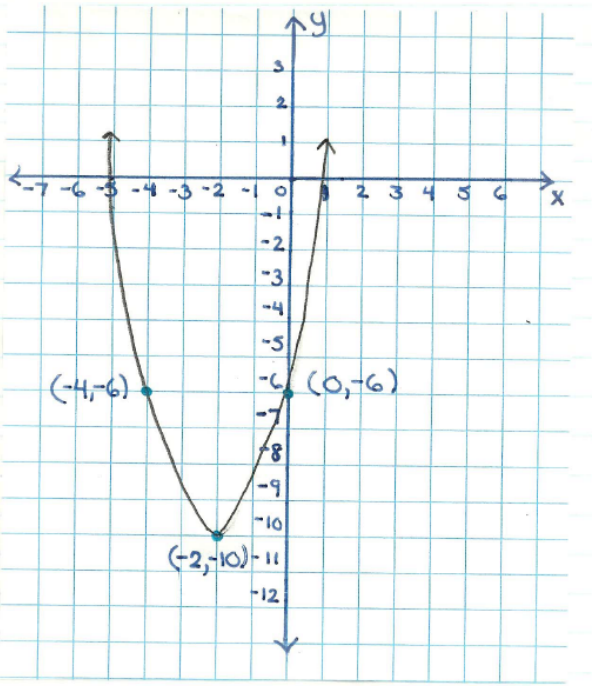
$=4-8-6$

$=-4-6$

$=-10$

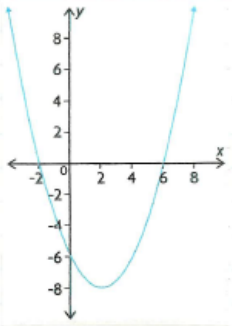
Vertex: $(-2,-10)$

iii)



11. {a,d} Determine the equation of the quadratic function that defines each parabola.

a)



x-intercepts: $x=-2$ & $x=6$ (r, s)

Vertex: $(2,-8)$ (x, y)

$y = a(x-r)(x-s)$

$y = a(x--2)(x-6)$

$y = a(x+2)(x-6)$

To find a:

$-8 = a(2+2)(2-6)$

$-8 = a(4)(-4)$

$\frac{-8}{-16} = \frac{-16a}{-16}$

$\frac{1}{2} = a$

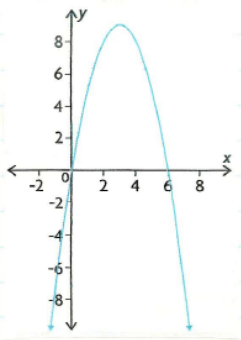
$\Rightarrow \quad y = \frac{1}{2}(x+2)(x-6)$

$y = \left(\frac{1}{2}x+1\right)(x-6)$

$y = \frac{1}{2}x^2-3x+1x-6$

$y = \frac{1}{2}x^2-2x-6$

d)



x-intercepts: $x = 0$ & $x = 6$
(r, s)

Vertex: $(3, 9)$
(x, y)

$y = a(x-r)(x-s)$
$y = a(x-0)(x-6)$
$y = a(x)(x-6)$

To find a:
$9 = a(3)(3-6)$
$9 = a(3)(-3)$
$\frac{9}{-9} = \frac{-9a}{-9}$
$-1 = a$

$\Rightarrow y = -1(x)(x-6)$
$y = -1x(x-6)$
$y = -1x^2 + 6x$